Centre of Mass

2010

1. **The centre of mass of a system of three particle of masses 1 g, 2g and 3g is taken as the origin of a coordinate system. The position vector of a fourth particle of mass 4g such that the centre of mass of the four particle system lies at the point (1, 2, 3) is $\alpha(\hat{i}+2\hat{j}+3\hat{k})$, where α is a constant. The value of α is**

   a) $\frac{10}{3}$ b) $\frac{5}{2}$ c) $\frac{1}{2}$ d) $\frac{2}{5}$

2008

2. **An object placed on a ground is in stable equilibrium. If the object is given a slight push then initially the position of centre of gravity**

   a) Moves nearer to ground b) Rises higher above the ground
   c) Remains as such d) May remain at same level

3. **Two bodies of different masses of 2kg and 4kg moving with velocities $2ms^{-1}$ and $10ms^{-1}$ towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass?**

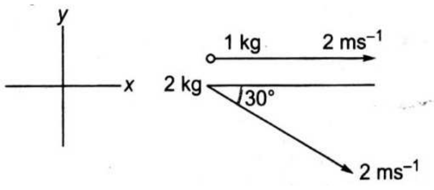
   a) $5ms^{-1}$ b) $6ms^{-1}$ c) $8ms^{-1}$ d) Zero

2006

4. **A small disc of radius 2cm is cut from a disc of radius 6cm. If the distance between their centres is 3.2cm what is the shift in the centre of mass of the disc**

   a) 0.4 cm b) 2.4 cm c) 1.8cm d) 1.2cm

5. **Find the velocity of centre of mass of the system shown in the figure.**



a) $\left(\frac{2+2\sqrt{3}}{3}\right)\hat{i}-\frac{2}{3}\hat{j}$

b) $4\hat{i}$

c) $\left(\frac{2+2\sqrt{3}}{3}\right)\hat{i}-\frac{1}{3}\hat{j}$

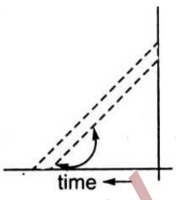
d) None of these

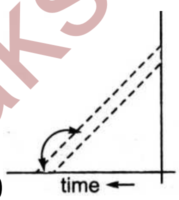
6. **A straight rod of length L one of its ends at the origin and the other at x = Lx. If the mass per unit length of the rod is given by Ax here A is constant, where is its mass centre**

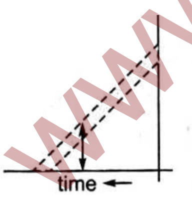
a) L/3 b) L/2 c) 2L/3 d) 3L/4

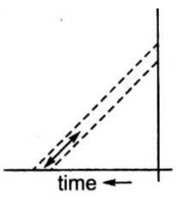
2005

7. **A ladder is leaned against a smooth wall and it is allowed to slip on a frictionless floor. Which figure represents the track of its centre of mass?**

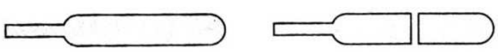
a) 

b) 

c) 

d) 

8. **A cricket bat is at the location of its centre of mass as shown then**



a) The two pieces will have the same mass

b) The bottom piece will have larger mass

c) The handle piece will have larger mass

d) Mass of handle piece is double the mass of bottom piece

2004

9. Consider a system of two particles having masses and m_2. If the particle of mass m_1 is pushed towards the mass centre of particle through a distance d, by what distance would the particle of mass m_2 moves so as to keep the mass centre of particle at the original position?

a) $\frac{m_1}{m_1+m_2}d$ b) $\frac{m_1}{m_2}d$

c) d d) $\frac{m_2}{m_1}d$

10. The centre of mass of a system of two particles divides the distance between them

a) In inverse ratio of square of masses of particles

b) In direct ratio of square of masses of particles

c) In direct ratio of square of masses of particle

d) In inverse ratio of masses of particles

Key

1) **b**	2) **b**	3) **d**	4) **a**	5) **a**	6) **b**	7) **c**	8) **d**	9) **b**	10) **c**

Hints

1. Then coordinates (x, y, z) of masses 1g, 2g, 3g and 4g are

$(x_1 = 0, y_1 = 0, z_1 = 0)\ (x_2 = 0, y_2 = 0, z_2 = 0)$

$(x_3 = 0, y_3 = 0, z_3 = 0)\ (x_4 = \alpha, y_4 = 2\alpha, z_4 = 3\alpha)$

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$x_{CM} = \frac{4\alpha}{1+2+3+4}$$

$$= \frac{4\alpha}{10}$$

$$1 = \frac{4\alpha}{10}$$

$$\Rightarrow \alpha = \frac{5}{2}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$2 = \frac{4\times 2\alpha x}{10}$$

$$\alpha = \frac{20}{8} = \frac{5}{2}$$

$$z_{CM} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + m_4z_4}{m_1 + m_2 + m_3 + m_4}$$

$$3 = \frac{4\times 3\alpha}{10}$$

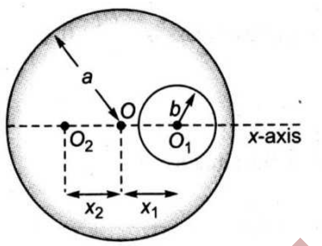
$$\alpha = \frac{5}{2}$$

2. In stable equilibrium, the centre of gravity of object lies at minimum height from ground. As the object is given a slight push, its centre of gravity rises because it comes in unstable equilibrium

3. As gravitational forces of attraction are mutual, centre of mass is not affected (in the absence of external force). Velocity of centre of mass is zero

4. Let radius of complete disc is a, and that of small disc is b. Also let centre of mass now shifts to O_2 at a distance x_2 from original centre

The position of new centre of mass is given by



$$X_{CM} = \frac{-\sigma\pi b^2 x_1}{\sigma\pi a^2 - \sigma\pi b^2}$$

Here a = 6cm, b = 2cm, $x_1 = 3.2cm$

Hence, $X_{CM} = \frac{-\sigma\times\pi(2)^2\times3.2}{\sigma\times\pi\times(6)^2 - \sigma\times\pi\times(2)^2}$

$= \frac{12.8}{32\pi}$

= - 0.4cm

5. Here $m_1 = 1kg$, $v_1 = 2\hat{i}$

$m_2 = 2kg$, $v_2 = 2\cos 30^0\hat{i} - 2\sin 30^0\hat{j}$

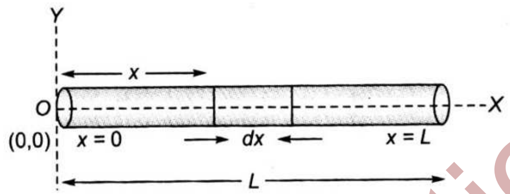
$$V_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{1\times 2i + 2(2\cos 30^0\hat{i} - 2\sin 30^0\hat{j})}{1+2}$$

$$= \frac{2\hat{i} + 2\sqrt{3}\hat{i} - 2\hat{j}}{3}$$

$$= \left(\frac{2+2\sqrt{3}}{3}\right)\hat{i} - \frac{2}{3}\hat{j}$$

6. Let of the mass of an element of length dx of the rod located at a distance x away from left end is $\frac{M}{L}dx$. The x-coordinate of the centre of mas is given by



$$X_{CM} = \frac{1}{M}\int x\ dm$$

$$\frac{1}{M}\int_0^L x\left(\frac{M}{L}dx\right)$$

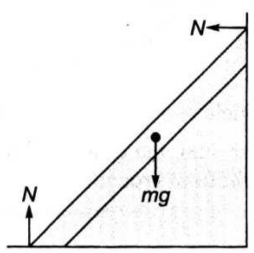
$$= \frac{1}{L}\left(\frac{x^2}{2}\right)_0^L = \frac{L}{2}$$

The y-coordinate is $Y_{CM} = \frac{1}{M}\int y\ dn = 0$

And similar, $Z_{CM} = 0$

Hence the centre of mass is at $\left(\frac{L}{2}, 0, 0\right)$ or at the middle point of the rod, i.e. at $\frac{L}{2}$.

7. The track of the centre of mass of ladder will be as shown in the figure



9. The system of two given particles of masses m_1 and m_2 are shown in figure

Initially the centre of mass $r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

When mass m_1 moves towards centre of mass by a distance d. then let mass m_2 moves a distance d' away from CM to keep

So, $r_{CM} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2}$

Equation eqs (i) and (ii), we get

$$\frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2}$$

$$\Rightarrow -m_1 d + m_2 d' = 0 \Rightarrow d' = \frac{m_1}{m_2} d$$