# **Circular Motion**

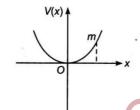
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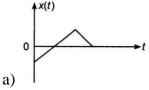
1. A particle moves in a circle of radius 5 cm with constant speed and time period  $0.2\pi\,s$  . The acceleration of the particle is

- a)  $25m/s^2$
- b)  $36m/s^2$
- c)  $5m/s^2$
- d)  $15m/s^2$

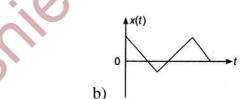
2. A particle of mass m is released from rest and follows a parabolic path as shown.

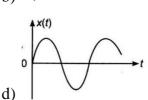
Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time?











- 4. A ball of mass 0.12kg is being whirled in a horizontal circle at the end of string 0.5m long. It is capable of making 231 revolutions in one minute. The breaking tension of the string is
  - a) 3 N

c)

- b) 15.1N
- c) 31.5N
- d) 35.1N
- 6. The motor of an angle is rotating about its axis with an angular velocity of 100 rev/m. It comes to rest in 15s, after being switched off. Assuming constant angular deceleration. What are the numbers of revolutions made by it before coming to rest?

c) 32.6

d) 15.6

a) 12.5

9.

top of the hill is between

a)  $14ms^{-1}$  and  $15ms^{-1}$ 

c)  $16ms^{-1}$  and  $17ms^{-1}$ 

b) 40

7. A stone of mass m is tied to a string and is moved in a vertical circle of radiu								
	making n rev/min. The total tension in the string when the stone is at the lowest point is  a) mg  b) $m(g + \pi nr^2)$ c) m $(g + nr)$ d) $m\left\{g + \frac{\pi^2 n^2 r}{900}\right\}$							
	is							
	a) mg	b) $m(g + \pi nr^2)$	c) m (g + nr)	$d) m \left\{ g + \frac{\pi^2 n^2 r}{900} \right\}$				
6.	A string is wo	ound round the rim of a m	nounted flywheel of ma	ss 20kg and radius				
	20cm. A steady pull of 25N is applied on the cord. Neglecting friction and mass of the							
	string, the an	gular acceleration of the		<b>()</b>				
	a) $50s^{-2}$	b) $25s^{-2}$	c) $12.5s^{-2}$	d) 6.25s <sup>-2</sup>				
7.								
	rest in 1min. The angular retardation in $rad / s^2$ is							
	a) $\frac{\pi}{2}$	b) $\frac{\pi}{4}$	c) $\frac{\pi}{6}$	d) $\frac{\pi}{8}$				
8.	A car is movi	ng in a circular horizonta	l track of radius 10.0 i	n with a constant speed				
	of $10.0ms^{-1}$ . A	plumb bob is suspended	from the roof of the ca	ar by a light rigid rod of				
length 1.00 m. The angle made by the rod with the track is $(g = 10ms^{-2})$								
	a) Zero	b) 30°	c) 45°	d) 60°				
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		4.						

10. Assertion: A body of mass 1kg is making 1rps in a circle of radius 1m. Centrifugal force acting on it is  $4\pi^2 N$ .

A roller coaster is designed such that riders experience 'weightlessness' as they go

round the top of a hill whose radius of curvature is 20m. The speed of the car at the

b)  $15ms^{-1}$  and  $16ms^{-1}$ 

d)  $13ms^{-1}$  and  $14ms^{-1}$ 

Reason: Centrifugal force is given by $F = \frac{mv^2}{r}$							
a) Both assertion and reason are true and reason is the correct explanation of assertion.							
b) Both assertion a	and reason are true but	reason is not the correct e	xplanation of assertion.				
c) Assertion is true	e but reason is false.						
d) Both assertion a	and reason are false.						
A particle is mov	ing along a circular p	eath of radius 4m with a	uniform speed $5ms^{-1}$ .				
What will be the	average acceleration	when the particle comple	etes half revolution?				
a) Zero	b) 10ms <sup>-2</sup>	c) $10\pi ms^{-2}$	d) $\frac{10}{\pi} ms^{-2}$				
A car of mass m	moves in a horizontal	circular path of radius r					
speed is $v m s^{-1}$ and is increasing at a rate of $a m s^{-2}$ . Then the acceleration of the car is							
a) $\frac{v^2}{r}$	b) a	$c) \sqrt{a^2 + \left(\frac{v^2}{r}\right)^2}$	d) $\sqrt{u + \frac{v^2}{r}}$				
A body of mass 1kg is rotating in a vertical circle of radius 1m. What will be the							
difference in its kinetic energy at the top and bottom of the circle (take $g = 10ms^{-2}$ )							
a) 10 J	b) 20 J	c) 30 J	d) 50 J				
A particle moves through angular displacement $ heta$ on a circular path of radius r. The							
linear displacement will be							
a) $2r\sin(\theta/2)$	b) $2r\cos(\theta/2)$	c) $2r \tan(\theta/2)$	d) $2r \cot(\theta/2)$				

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11.

**12.** 

13.

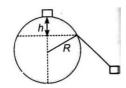
**14.** 

0.5. What should be its angular speed in  $rads^{-1}$  if it is not to slip from the surface?  $(g = 9.8 ms^{-2})$ a) 5 b) 10 c) 0.1 d) 0.7

15. A body moves along a circular path of radius 10m and the coefficient of friction is

16.	One end of a string	g of length l is connected	to a particle of mass i	m and the other to a			
	small peg on a smooth horizontal table. If the particle moves in a circle with speed v,						
	the net force on the	e particle (directed towa	rds the centre) is				
	a) T	b) $T - \frac{mv^2}{l}$	c) $T + \frac{mv^2}{l}$	d) Zero			
17.	If $a_r$ and $a_t$ repres	ent radial and tangentia	l accelerations, the mo	otion of a particle			
	will be uniformly o	ircular if					
	a) $a_r = 0$ and $a_t = 0$	b) $a_r = 0$ but $a_t \neq 0$	c) $a_r \neq 0$ but $a_t = 0$	d) $a_r \neq 0$ and $a_t \neq 0$			
			<b>(</b>	•			
200	5		(0)	•			
18.							
	constant speed. If the stone makes 22 revolutions in 44s, what is the magnitude and						
	direction of acceleration the centre?						
	a) $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre						
	b) $\pi^2 ms^{-2}$ and direction along the radius towards from centre						
	c) $\pi^2 ms^{-2}$ and direction along the radius towards the centre						
	d) $\pi^2 ms^{-2}$ and direct	tion along the tangent to t	he circle				
19.	What determines the nature of the path followed by the particle?						
	a) Speed	b) Velocity	c) Acceleration	d) Both b and c			
20.	A ball of mass 0.25	kg attached to the ends	of a string of length 1.	96m is rotating in a			
	horizontal circle. T	he string will break, if t	ension is more than 25	5N. What is the			
	maximum velocity	with which the ball can	be rotated?	the centre  om centre  de centre  cle  d by the particle?  Acceleration d) Both b and c  tring of length 1.96m is rotating in a  n is more than 25N. What is the  otated?  Oms <sup>-1</sup> d) 14ms <sup>-1</sup>			
	a) $3ms^{-1}$	b) 5ms <sup>-1</sup>	c) 9ms <sup>-1</sup>	d) $14ms^{-1}$			
21.	When a body move	es with constant speed in	a circular path, then				
	a) Work done will b	e zero	b) Acceleration will	be zero			
	c) No force acts on the body  d) Its velocity remains constant						

22.	A stone of mass m tied	d to a string of len	gth l is rotated a circle wi	th the other end of			
	the string as the centre. The speed of the stone is v. If the string breaks, the stone will						
	a) Move towards the ce	entre	b) Move away from t	he centre			
	c) Move along tangent		d) Stop				
200	4						
23.	The direction of the angular velocity vector is along						
	a) The tangent to the ci	rcular path	b) The inward radius	O			
	c) The outward radius		d) The axis of rotation	n			
24.	In an orbital motion,	the angular mome	entum vector is				
	a) Along the radius vec	tor	b) Parallel to the line	b) Parallel to the linear momentum			
	c) In the orbital plane		d) Perpendicular to the	d) Perpendicular to the orbital plane.			
200: 25.	A particle moves a lor		$(\pi)$				
			cle is $80ms^{-1}$ at the end of tangential acceleration is	the second			
		) $40ms^{-2}$	c) $40\pi ms^{-2}$	d) $640\pi ms^{-2}$			
26.	Force responsible for	the circular motion	on of the body is				
	a) Centripetal Force by	) Centrifugal Force	c) Gravitational Forc	e d) None of these			
27.	A cyclist moving at a	speed of 20ms <sup>-1</sup> ta	kes a turn, if the doubles	his speed then			
	chance of overturn						
	a) Is doubled by	) Is halved	c) Becomes four times	d) Becomes ¼ times			
28.	A particle originally at rest at the highest point of a smooth circle in a vertical plane,						
	is gently pushed and starts sliding along the circle in a vertical plane, is gently						
	pushed and starts sliding along the circle. It will leave the circle at a vertical distance						
	h below the highest point such that						



- a) h = 2R
- b)  $h = \frac{R}{2}$

- c) h = R
- $d) h = \frac{R}{3}$

#### Key

1) <b>c</b>	2) <b>d</b>	3) <b>d</b>	4) <b>a</b>	5) <b>d</b>	6) <b>c</b>	7) <b>a</b>	8) <b>c</b>	9) <b>a</b>	10) <b>a</b>
11) <b>d</b>	12) <b>c</b>	13) <b>b</b>	14) <b>a</b>	15) <b>d</b>	16) <b>a</b>	17) <b>c</b>	18) <b>c</b>	19) <b>d</b>	20) <b>d</b>
21) <b>a</b>	22) <b>c</b>	23) <b>d</b>	24) <b>d</b>	25) <b>b</b>	26) <b>a</b>	27) <b>d</b>	28) <b>d</b>		

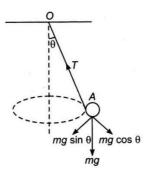
### Hints

1.  $r = 5cm = 5 \times 10^{-2} m$  and  $T = 0.2\pi s$ 

$$a = r\omega^2 = \frac{4\pi^2}{T^2}r = \frac{4\times\pi^2\times5\times10^{-2}}{(0.2\pi)^2} = 5ms^{-2}$$

- 2. Concept
- $3. \qquad F = \frac{mv^2}{r}$

Where, m is mass, v the velocity and r the radius



Since,  $v = r\omega$ 

$$F = mr\omega^2$$

But, 
$$m = 0.1 \text{kg}$$
,  $r = 0.5 \text{m}$ ,

$$\omega = 231 rpm = \frac{2\pi \times 231}{60} rad s^{-1} = 24.2 \text{ rad/s}$$

$$F = 0.12 \times 0.5 \times (24.2)^2 = 35.1N$$

4. 
$$0 = \omega_0 - \alpha t$$

$$\alpha = \frac{\omega_0}{t} = \frac{(100 \times 2\pi)/60}{15} = 0.6 rad/s^2$$

Now, 
$$\theta = \frac{\omega_0^2}{2\alpha}$$

Or 
$$\theta = \frac{\left(\frac{100 \times 2\pi}{60}\right)^2}{2 \times 0.7} = 78.25 rad$$

Number of rotations

$$n = \frac{\theta}{2\pi} = 12.5$$

5. 
$$T_{net} = \frac{mv^2}{r} + mg = mr\omega^2 + mg = mg\left(\frac{2\pi n}{60}\right)^2 + mg = m\left[\frac{\pi^2 n^2 r}{900} + g\right]$$

6. 
$$m = 20$$
kg

$$R = 20cm = \frac{1}{5}$$

But, 
$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 20 \times \frac{1}{5}$$

$$I = 0.4kg - m^2$$

Angular acceleration 
$$\alpha = \frac{\tau}{I} = \frac{FR}{I} = \frac{25 \times \frac{1}{5}}{0.4} = 12.5 s^{-2}$$

7. 
$$\omega = \omega_0 + \alpha t$$

$$\omega_0 = 900 rpm = \frac{(2\pi \times 900)}{60} rad / s$$

$$\omega = 0$$
 and  $t = 60$ 

$$0 = \frac{2\pi \times 900}{60} + \alpha \times 60$$

$$\alpha = \frac{2\pi \times 900}{60 \times 60} = \frac{\pi}{2}$$

8. 
$$\tan \theta = \frac{v^2}{rg} \Rightarrow \tan \theta = \frac{(10)^2}{10 \times 10} = 1$$

$$\Rightarrow \theta = 45^{\circ}$$

$$9. Mg - N = M \frac{v^2}{R}$$

For weightlessness, N = 0

$$\therefore \frac{Mv^2}{R} = Mg$$

Where R is the radius of curvature and v is the speed of car

Therefore, 
$$v = \sqrt{Rg}$$

But, R = 20m, 
$$g = 10.0 ms^{-2}$$

$$v = \sqrt{20 \times 10.0} = 14.14 ms^{-2}$$

But, R =20m, 
$$g = 10.0ms^{-2}$$
  
 $v = \sqrt{20 \times 10.0} = 14.14ms^{-2}$   
10.  $F = \frac{mv^2}{r} = \frac{n(r\omega)^2}{r} = mr\omega^2 = mr(2\pi v)^2 = 4\pi^2 mrv^2$ 

$$F = 4\pi^2 \times 1 \times 1 \times 1^2 = 4\pi^2 N$$

11. Change in velocity

$$\Delta v = [5 - (-5)]ms^{-1} = 10ms^{-1}$$

Time taken to complete the half revolution is

$$t = \frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi \text{second}$$

Average acceleration 
$$a_v = \frac{\Delta v}{t} = \frac{10}{\pi} ms^{-2}$$

12. Radial acceleration 
$$a_r = \frac{v^2}{r}$$

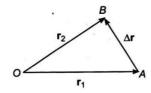
Tangential acceleration  $a_t = a$ 

 $\therefore \text{ Resultant acceleration } a' = \sqrt{a_r^2 + a_t^2 + 2a_r a_t \cos \theta}$ 

But 
$$\theta = 90^{\circ}$$

And 
$$a' = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$$

- 13. Difference in kinetic energy =  $\frac{1}{2}m[5gr gr] = 2gmr = 2 \times 10 \times 1 \times 1 = 20J$
- 14.  $\Delta r = r_2 r_1$ ; where  $r_2 = r_1 = r$



$$\Delta r = \sqrt{r_2^2 + r_1^2 - 2r_2r_1\cos\theta} = 2r\sin\frac{\theta}{2}$$

15. 
$$mr\omega^2 = \mu mg$$

Or 
$$r\omega^2 = \mu g$$

Or 
$$\omega = \sqrt{\frac{0.5 \times 9.8}{10}} = 0.7 \, rad \, s^{-1}$$

- 16. Concept
- 17. a) if  $a_r = 0$  and  $a_t = 0$  then motion is uniform translatory
  - b) if  $a_r = 0$  but  $a_t \neq 0$  then motion is acceleration translatory
  - c) if  $a_r \neq 0$  and  $a_t \neq 0$ , then motion is a uniform circular
  - d) if  $a_r \neq 0$  and  $a_t \neq 0$  then motion is a non uniform circular

18. 
$$a = r\omega^2 = r\left(\frac{2\pi n}{t}\right)^2 = r \times \frac{4\pi^2 n^2}{t^2} = \frac{1 \times 4 \times \pi^2 \times (22)^2}{(44)^2} = \pi^2 m s^{-2}$$

This acceleration is directed along radius of circle

19. Concept

$$20. \quad F = \frac{mv^2}{r}$$

Or 
$$v^2 = \frac{Fr}{m}$$

$$\therefore v^2 = \frac{25 \times 1.96}{0.25} = 196$$

Or 
$$v = \sqrt{196} = 14ms^{-1}$$

- 21. Concept
- 22. Concep
- Concept
- 24. Concept
- 25. The tangential acceleration

$$a_T = r\alpha$$

But from,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\therefore v = \frac{196}{0.25} = 196$$
Or  $v = \sqrt{196} = 14ms^{-1}$ 
Concept
Concept
Concept
The tangential acceleration
$$a_T = r\alpha$$
But from,
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega_0 = 0, \ \omega = \frac{v}{r} = \frac{80}{20/\pi} = 4\pi \ rad \ s^{-1}$$

$$\theta = 2 \times 2\pi \ rad$$

$$\theta = 2 \times 2\pi \, rad$$

$$\alpha = \frac{\omega^2}{20} = \frac{(4\pi)^2}{2 \times (2 \times 2\pi)} = \frac{16\pi^2}{8\pi} = 2\pi$$

$$\therefore a_T = r\alpha = \frac{20}{\pi} \times 2\pi = 40ms^{-2}$$

26. Concept

$$27. \quad F = \frac{mv^2}{r}$$

$$\Rightarrow F \propto v^2$$

Hence chance of overturning becomes  $\frac{1}{4}$  times

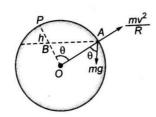
28. From law of conservation of energy,

$$\therefore PE = KE$$

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{2gh}$$
 .....(i)

Also the horizontal component of force is equal centrifugal force



$$\therefore mg\cos\theta = \frac{mv^2}{R} \dots (ii)$$

From eq (i)

$$v = \sqrt{2gh}$$

$$\therefore mg\cos\theta = \frac{2mgh}{R} \dots (iii)$$

From  $\triangle AOB$ ,

$$\cos\theta = \frac{2R - h}{R}$$

$$\Rightarrow mg\left(\frac{R-h}{R}\right) = \frac{2mgh}{R} \Rightarrow 3h = R$$

$$\Rightarrow h = \frac{R}{3}$$