Vectors

2011

1. If vectors $\hat{i}-3\hat{j}+5\hat{k}$ and $\hat{i}-3\hat{j}-a\hat{k}$ are equal vectors, then the value of a is

a) 5 b) 2 c) -3 d) 4

2010

2. If a + b = c and a + b = c, then the angle included between a and b is

a) 90^0 b) 180^0 c) 120^0 d) Zero

3. Three equal masses of 1kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}m$ from each of the vertices of the triangle. The force, in Newton, acting on the mass of 2kg is

a) 2 b) 1 c) 1 d) Zero

4. Find the torque of a force $F=3\hat{i}+2\hat{j}+\hat{k}$ acting at the point $r=8\hat{i}+2\hat{j}+3\hat{k}$

a) $14\hat{i}-38\hat{j}+16\hat{k}$ b) $4\hat{i}+4\hat{j}+6\hat{k}$ c) $-14\hat{i}+38\hat{j}-16\hat{k}$ d) $-4\hat{i}-17\hat{j}+22\hat{k}$

5. A variable force, given by the two dimensional vector $F=(3x^2\hat{i}+4\hat{j})$, acts on a particle. The force is in Newton and x is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2, 3) to (3, 0)? (The coordinates are in metres)

a) – 7 J b) Zero c) +7J d) 19J

6. The centre of mass of a system of three particles of masses 1 g, 2g and 3g is taken as the origin of a coordinates system. The position vector of a fourth particle of mass 4g such that the centre of mass of the four particle system lies at the point (1, 2, and 3) is $\alpha(\hat{i}+2\hat{j}+3\hat{k})$, where it is constant. The value of α is

a) 10/3 b) 5/2 c) 1/2 d) 2/5

2009

7. If a_1 and a_2 are two non-collinear unit vectors and if $|a_1 + a_2| = \sqrt{3}$, then the value of $(a_1 - a).(2a_1 + a_2)$ is

a) 2 b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) 1

8. There are N coplanar vectors each of magnitude V. Each vector is inclined to the preceding vector at angle $\frac{2\pi}{N}$. What is the magnitude of their resultant?

a) $\frac{V}{N}$ b) V c) Zero d) $\frac{N}{V}$

2008

9. The value of P so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + P\hat{j} + 5\hat{k}$ are coplanar should be

a) 16 b) -4 c) 4 d) -8

10. Two forces of 12N and 8N act upon a body. The resultant force on the body has a maximum value of

a) 4N b) Zero c) 20N d) 8N

11. The condition under which vectors (a + b) and (a – b) should be at right angles to each other is

a) $a \neq b$ b) a.b = 0 c) $|a| = |b|$ d) a. b = 1

12. A car travels 6km towards North at an angle of 45^0 to the East and then travels distance of 4kg towards North at an angle 135^0 to East. How far is the point from the starting point? What angle does the straight line joining its initial and final positions makes with the East?

a) $\sqrt{50}km$ and $\tan^{-1}$ b) 10 km and $\tan^{-1}(\sqrt{5})$

c) $\sqrt{52}$ km and $\tan^{-1}(5)$ d) $\sqrt{52}$ km and $\tan^{-1}(\sqrt{5})$

13. A train of 150 m length is going towards North direction at a speed of $10ms^{-1}$. A parrot flies at a speed of $5ms^{-1}$ towards South direction parallel to the railway track. The time taken by the parrot to cross the train is equal to

a) 12s b) 8s c) 15s d) 10s

14. Rain is falling vertically downwards with a velocity of $4kmh^{-1}$. A man walks in the rain with a velocity of $3kmh^{-1}$. The raindrops will fall on the man with a velocity is

a) $1kmh^{-1}$ b) $3kmh^{-1}$ c) $4kmh^{-1}$ d) $5kmh^{-1}$

15. A proton in a cyclotron changes its velocity from $30kms^{-1}$ North to $40kms^{-1}$ East in 20s. What is the magnitude of average acceleration during this time?

a) $2.5kms^{-2}$ b) $12.5kms^{-2}$ c) $22.5kms^{-2}$ d) $32.5kms^{-2}$

2006

16. A police jeep is chasing with velocity of $45kmh^{-1}$, a thief in another jeep moving with velocity $153kmh^{-1}$. Police fires a bullet with muzzle velocity of $180ms^{-1}$. The velocity with which will strike the car of the thief, is

a) $150ms^{-1}$ b) $27ms^{-1}$ c) $450ms^{-1}$ d) $250ms^{-1}$

2005

17. Minimum of unequal vectors which can gives zero resultant are

a) Two b) Three c) Four d) More than four

18. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

19. The vectors from origin to the points A and B are $A=3\hat{i}-6\hat{j}+2\hat{k}$ and $B=2\hat{i}+\hat{j}-2\hat{k}$ respectively. The area of the triangle OAB is

a) $\frac{5}{2}\sqrt{17}$ b) $\frac{2}{5}\sqrt{17}$ c) $\frac{3}{5}\sqrt{17}$ d) $\frac{5}{3}\sqrt{17}$

Key

1) **c**	2) **d**	3) **d**	4) **d**	5) **c**	6) **b**	7) **c**	8) **c**	9) **b**	10) **c**
11) **c**	12) **c**	13) **d**	14) **d**	15) **a**	16) **a**	17) **b**	18) **c**	19) **a**	

Hints

1. Given vectors

$A = \hat{i} - 3\hat{j} + 5\hat{k}$

$B = \hat{i} - 3\hat{j} - a\hat{k}$

According to problem both vectors are equal then A = B, so that the value of a =-5

2. We have a + b = c and c = a + b

$\Rightarrow c = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\Rightarrow a + b = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

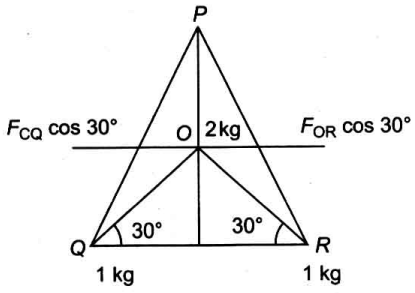
$\Rightarrow a^2 + b^2 + 2ab = a^2 + b^2 + 2ab\cos\theta$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0^0$

3. Given, OP = OQ = OR = $\sqrt{2}m$

The gravitational force on the mass 2kg due to the 1kg mass at P is

$F_{OP} = G\frac{2\times 1}{(\sqrt{2})^2} = G$ along OP



Similarly, $F_{OQ} = G_1\frac{2\times 1}{(\sqrt{2})^2} = G_1$ along OQ and

$F_{OR} = G_1 \frac{2\times1}{(\sqrt{2})^2} = G_1$ along OR

$F_{OQ}\cos 30^0$ and $F_{OR}\cos 30^0$ are equal and acting in opposite directions, thus they cancel out.

Hence, the resultant force on the 2kg mass at O, is

$F = F_{OP} = (F_{OQ}\sin 30^0 + F_{OR}\sin 30^0)$

$= G_1 - \left(\frac{G_1}{2} + \frac{G_1}{2}\right)$

= 0

4. Torque of the force, $\tau = r \times F$

So, $\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 2 & 3 \\ -3 & 2 & 1 \end{vmatrix}$

$= \hat{i}(2-6) - \hat{j}(8+9) + \hat{k}(16+6)$

$= -4\hat{i} - 17\hat{j} + 22\hat{k}$

5. Given two dimensional force

$F = 3x^2\hat{i} + 4\hat{j}$

$r = x\hat{i} + y\hat{j}$

$dr = dx\hat{i} + dy\hat{j}$

Kinetic energy = work done

$W = \int F.dr$

$= \int_{(2,3)}^{(3,0)} (3x^2\hat{i} + 4\hat{j}).(dx\hat{i} + dy\hat{j})$

$= \int_2^3 (3x^2dx + 4dy)$

$= [x^3]_2^3 + 4[y]_3^0 = (27-8) + 4(-3)$

= 19 -12 = 7J

6. The coordinates (x, y, z) of masses 1g, 2g, 3g and 4g are $(x_1 = 0, y_1 = 0, z_1 = 0)$,

$(x_2 = 0, y_2 = 0, z_2 = 0)$

$\Rightarrow x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$= \frac{4\alpha}{1+2+3+4}$

$= \frac{4\alpha}{10}$

Hence, $\frac{4\alpha}{10} = 1$

$\Rightarrow \alpha = \frac{5}{2}$

$\Rightarrow y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4} = \frac{8\alpha}{10} = 2$

$\Rightarrow \alpha = \frac{5}{2}$

$\Rightarrow z_{CM} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + m_4z_4}{m_1 + m_2 + m_3 + m_4}$

$= \frac{12\alpha}{10} = 3 \Rightarrow \alpha = 5/2$

7. Since, a_1 and a_2 are non=collinear

$\therefore a_1 = a_2 = 1$

And $|a_1 + a_2| = \sqrt{3}$

$\Rightarrow a_1^2 + a_2^2 + 2a_1a_2\cos\theta = (\sqrt{3})^2$

$\Rightarrow 1+1+2\cos\theta = 3 \Rightarrow \cos\theta = \frac{1}{2}$

Now $(a_1 + a_2).(2a_1 + a_2)$

$= 2a_1^2 - a_2^2 - a_1a_2\cos\theta = 2 - 1 - \frac{1}{2} = \frac{1}{2}$

8. Since each of N-coplanar vectors is inclined at $\frac{2\pi}{N}$ to the preceding hence, they will form a closed polygon. Therefore, their resultant must be zero

9. For coplanarity

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & P & 5 \end{vmatrix} = 0$$

Or 2(10 +3P) +1(5 + 9) + 1 (P – 6) =

Or 20 + 6P + 5 + 9 + P – 6 = 0

Or 7P + 34 – 6 = 0

Or 7P + 28 = 0

Or 7P = -28

$\Rightarrow P = -\frac{28}{7} = -4$

10. When the two forces of 12N 8N act upon a body, the resultant force on the body has maximum value when resultant force

= 12N + 8N = 20N

11. The dot product of two vectors should be equal to zero is (a + b). (a – b) =0

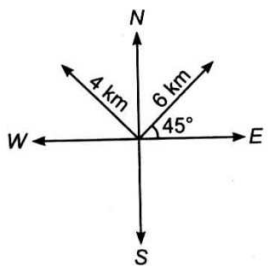
$\Rightarrow a^2 - b^2 = 0$

$\Rightarrow |a| = |b|$

12. Net movement along X-direction

$S_x = (6-4)\cos 45^0 \hat{i}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} km$



Net movement along Y-direction

$S_y = (6+4)\sin 45^0\,\hat{j}$

$= 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}km$

Net movement from starting point

$|S| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2}$

$= \sqrt{52}km$

Angle which resultant makes with the east direction

$\tan\theta = \frac{y-component}{x-component}$

$= \frac{5\sqrt{2}}{\sqrt{2}}$

$\theta = \tan^{-1}(5)$

13. Relative velocity of the parrot w.r.t the train

$= [10-(-5)]ms^{-1} = 15ms^{-1}$

Time taken by the parrot to cross the train

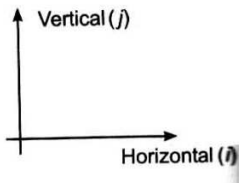
$\frac{150}{15} = 10s$

14. Relative velocity of man w.r.t rain

$v_{rm} = v_r - v_m$

$= 4\hat{j} - 3\hat{i}$

$= -3\hat{i} + 4\hat{j}$



Or $= 5kmh^{-1}$

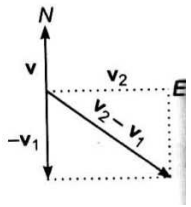
$|v_{rm}| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$

16. Change in velocity $= \sqrt{(40)^2 + (30)^2}$

$= 50 kms^{-1}$



Average acceleration $= \frac{50}{20}$

$= 2.5 kms^{-2}$

17. The relative velocity of thief's jeep with respect to police jeep = 153 – 45

$= 108 kmh^{-1}$

$= 108 \times \frac{5}{18} = 30 ms^{-1}$

Therefore, striking speed = relative speed of bullet with respect to thief's car

= 180 – 30 = $150 ms^{-1}$

18. By triangle law of vectors minimum three vectors are required to give zero resultant

19. Let $a = 2\hat{i} + 3\hat{j} + 8\hat{k}$

$b = 4\hat{i} - 4\hat{j} + \alpha\hat{k}$

$= -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Given $a \perp b$

$\Rightarrow a.b = 0$

$\Rightarrow (2\hat{i} + 3\hat{j} + 8\hat{k}).(-4\hat{i} + 4\hat{j} + \alpha\hat{k}) = 0$

$\Rightarrow -8 + 12 + 8\alpha = 0$

$\Rightarrow 8\alpha = -4$

$\therefore \alpha = -\frac{4}{8} = -\frac{1}{2}$