Gravitation

Newton's Law of Gravitation

2011

1. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

a) r_2/r_1 b) $(r_2/r_1)^2$ c) r_1/r_2 d) $(r_1/r_2)^2$

2. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest

a) At the instant just before the body hits the earth

b) It remains constant all through

c) At the instant just after the body is projected

d) At the highest position of the body

2010

3. One can easily weigh the earth by calculating the mass of the earth by using the formula (in usual notation)

a) $\frac{G}{g}R_E^2$ b) $\frac{g}{G}R_E^2$ c) $\frac{g}{G}R_E$ d) $\frac{G}{g}R_E^3$

4. Three equal masses of 1kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}m$ from each of the vertices of the triangle. The force, in Newton, acting on the mass of 2kg is

a) 2 b) $\sqrt{2}$ c) 1 d) Zero

5. **What will be the acceleration due to gravity at a depth d where g is acceleration due to gravity on the surface of earth?**

a) $\frac{g}{\left[1+\frac{d}{R}\right]^2}$ b) $g\left[1-\frac{2d}{R}\right]$ c) $\frac{g}{\left[1-\frac{d}{R}\right]^2}$ d) $g\left[1-\frac{d}{R}\right]$

6. **The height vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)**

a) 8R b) 9R c) 10R d) 20R

7. **A planet has same density and same acceleration due to gravity as of earth and universal gravity as of earth and universal gravitational constant G is twice of earth. The ratio of their radii is**

a) 1 : 4 b) 1: 5 c) 1 : 2 d) 3 : 2

8. **Two particle, initially at rest move towards each other under the effect of gravitational force of attraction. At the instant when their relative velocity is 3v where, v is the velocity of the slower particle, then the speed of the centre of mass of two given particle is**

a) 1v b) 2v c) 3v d) Zero

2009

9. **At what height above the surface of earth the value of acceleration due to gravity would be half of its value on the surface of earth? (Radius of the earth is 6400 km)**

a) 2561 km b) 2650 km c) 3200 km d) 9800 km

10. **Two particles of equal go round a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is**

a) $\sqrt{\left(\frac{GM}{2R}\right)}$ b) $\frac{1}{2R}\sqrt{\left(\frac{GM}{2R}\right)}$

c) $\frac{1}{2}\sqrt{\left(\frac{GM}{2R}\right)}$ d) $\sqrt{\left(\frac{4GM}{2R}\right)}$

2008

11. If the distance between the sun and the earth is increased by three times then attraction between two will

a) Remain constant b) Decrease by 63% c) Increase by 63% d) Decrease by 89%

2007

12. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

a) $\frac{F}{3}$ b) $\frac{F}{9}$ c) 3F d) F

2004

13. The mass of moon is 1% of mass of earth. The ratio of gravitational pull of earth on moon and that of moon on earth will be

a) 1 : 1 b) 1 : 10 c) 1 : 100 d) 2 : 1

Acceleration due to Gravity

2008

14. What will happen to the weight of the body at the south pole, if the earth stops rotating about its polar axis?

a) No changes b) Increases

c) Decreases but does not become zero d) Reduces to zero

15. Which of the following statements is/are true?

a) A clock when taken on a mountain can be made to give correct time if we change the length of pendulum suitably

b) An increase in value of g makes a clock go slow

c) It the length of a pendulum is increased, the clock becomes fast

d) A clock when taken to a deep mine or carried to the top of a mountain becomes slow

2007

16. The density of newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the plane would be

a) 2R b) 4R c) $\frac{1}{4}R$ d) $\frac{1}{2}R$

17. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B

a) 6m b) 2/3 m c) 2/9m d) 18m

2005

18. Imagine a new planet having the same density as that earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

a) $g' = 3g$ b) $g' = \frac{g}{9}$ c) $g' = 9g$ d) $g' = 27g$

2004

19. When you move from equator to pole, the value of acceleration due to gravity (g)

a) increases b) decreases

c) remains the same d) increases then decreases

20. A body weighs 500N on the surface of the earth. How much would it weigh half way below the surface of the earth?

a) 1000N b) 500N c) 250N d) 125N

Gravitational Energy

2010

21. The energy required to move a satellite of mass m from an orbit of radius 2R to 3R around earth of mass M is

a) $\frac{GMm}{12R}$ b) $\frac{GMm}{R}$ c) $\frac{GMm}{8R}$ d) $\frac{GMm}{2R}$

22. The escape velocity for the earth is v_e. The escape velocity for a planet whose radius is $\frac{1}{4}th$ the radius of earth and mass half that of the earth is

a) $\frac{v_e}{\sqrt{2}}$ b) $\sqrt{2}v_e$ c) $2v_e$ d) $\frac{v_e}{2}$

23. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

a) $\left(\frac{n}{n+1}\right)mgR$ b) $\left(\frac{n}{n-1}\right)mgR$ c) nmgR d) $\frac{mgR}{n}$

2008

24. A particle of mass 10g is kept on the surface of a uniform sphere of mass 100kg and radius 10cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take $G = 6.67\times10^{-11} Nm^2kg^{-2}$)

a) $13.34\times10^{-10} J$ b) $3.33\times10^{-10} J$ c) $6.67\times10^{-9} J$ d) $6.67\times10^{-10} J$

25. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

a) $\left[2G\frac{(m_1-m_2)}{r}\right]^{1/2}$ b) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$ c) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ d) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

2007

26. An asteroid of mass m is approaching earth, initially at a distance of $10R_e$ with speed v_i. It hits the earth with a speed v_f (R_e and M_e are radius and mass of earth), then

a) $v_f^2 = v_i^2 + \frac{2Gm}{M_e R}\left(1 - \frac{1}{10}\right)$

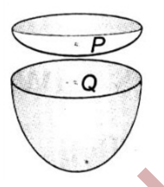
b) $v_f^2 = v_i^2 + \frac{2Gm_e}{R_e}\left(1 + \frac{1}{10}\right)$

c) $v_f^2 = v_i^2 + \frac{2Gm_e}{R_e}\left(1 - \frac{1}{10}\right)$

d) $v_f^2 = v_i^2 + \frac{2Gm}{R_e}\left(1 - \frac{1}{10}\right)$

2006

27. If a spherical shell is cut into two pieces along a chord as shown in figure and point P and Q have gravitational intensities I_P and I_Q respectively, then



a) $I_P > I_Q$ b) $I_P = I_Q = 0$ c) $I_P = I_Q \neq 0$ d) $I_P < I_Q$

2005

28. At what temperature, hydrogen molecules will escape from the earth's surface? (Take mass of hydrogen molecule $= 3.4 \times 10^{-26} kg$, **Boltzmann constant** $= 1.38 \times 10^{-23} JK^{-1}$, **radius of earth** $= 6.4 \times 10^6 m$ **and acceleration due to gravity** $= 9.8 ms^{-2}$**)**

a) 10 K b) $10^2 K$ c) $10^3 K$ d) $10^4 K$

29. There are two planets and the ratio of radius of the two planets is K but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity?

a) $(Kg)^{1/2}$ b) $(Kg)^{-1/2}$ c) $(Kg)^2$ d) $(Kg)^{-2}$

2004

30. **Escape velocity at surface of earth is** $11.2 kms^{-1}$ **. Escape velocity from a planet whose mass is the same as that of earth and radius ¼ that of earth, is**

a) $2.8 kms^{-1}$ b) $15.6 kms^{-1}$ c) $22.4 kms^{-1}$ d) $44.8 kms^{-1}$

2003

31. **The velocity with which a projectile must be fired so that its escapes earth's gravitation does not depend on**

a) Mass of the earth b) Mass of the projectile

c) Radius of the projectile's orbit d) Gravitational constant

Motion of Satellites and Kepler's Laws of Planetary Motion

2010

32. **By what percent the energy of a satellite has to the increased to shift it from an orbit of radius r to** $\frac{3}{2}r$?

a) 15% b) 20.3% c) 66.7% d) 33.33%

2009

33. **The period of revolution of an earth's satellite close to surface of earth is 90min. The time period of another satellite in an orbit at a distance of four times the radius of earth from its surface will be**

a) $90\sqrt{9}$ min b) 270 min c) 720 min d) 360 min

34. **A synchronous relay satellite reflects TV signals and transmits TV programme from one part of the world to the other because its**

a) Period of revolution is greater than the period of rotation of the earth about its axis.

b) Period of revolution is less than the period of rotation of the earth about its axis.

c) Period of revolution is equal to the period of rotation of the earth about its axis.

d) Mass is less than the mass of earth.

35. Two satellites of earth, S_1 and S_2, are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

a) The time period of S_1 is four times that of S_2

b) The potential energies of earth and satellite in the two cases are equal

c) S_1 and S_2 are moving with the same speed

d) The kinetic energies of the two satellites are equal

2007

36. The motion of planets in the solar system is an example of conservation of

a) Mass b) Momentum c) Angular Momentum d) Kinetic Energy

2006

37. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

a) 2 b) $\frac{1}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{2}$

38. The minimum energy required to launch a m kg satellite from earth's surface in a circular orbit at an altitude of 2R, R is the radius of earth, will be

a) 3mgR b) $\frac{5}{6}mgR$ c) 2mgR d) $\frac{1}{5}mgR$

39. A satellite is moving on a circular path of radius r around the earth has a time period T. If its radius slightly increases by Δr, the change in its time period is

a) $\frac{3}{2}\left(\frac{T}{r}\right)\Delta r$ b) $\left(\frac{T}{r}\right)\Delta r$ c) $\left(\frac{T^2}{r^2}\right)\Delta r$ d) None of these

40. The total energy of a satellite moving with an orbital velocity v around the earth is

a) $\frac{1}{2}mv^2$ b) $-\frac{1}{2}mv^2$ c) mv^2 d) $\frac{3}{2}mv^2$

41. The eccentricity of earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is

a) 2.507 b) 1.0339 c) 8.324 d) 1.000

2005

42. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. The orbital velocity of a satellite orbiting at an altitude of half of the radius, is

a) $\frac{3}{2}v_0$ b) $\frac{2}{3}v_0$ c) $\sqrt{\frac{2}{3}}v_0$ d) $\sqrt{\frac{3}{2}}v_0$

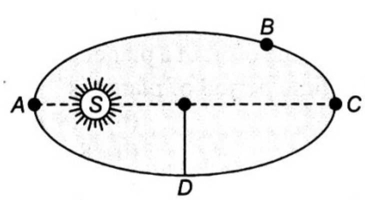
43. The earth revolves around the sun in one year. If distance between them becomes double, the new time period of revolution will be

a) $4\sqrt{2}years$ b) $2\sqrt{2}years$ c) 4 years d) 8 years

44. Two planets are revolving around the earth with velocities v_1 and v_2 in radii r_1 and $r_2 (r_1 > r_2)$ respectively. Then

a) $v_1 = v_2$ b) $v_1 > v_2$ c) $v_1 < v_2$ d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

45. A planet revolves in elliptical orbit around the sun. The linear speed of the planet will be 44maximum at



a) A b) B c) C d) D

2003

46. Assertion (A): The earth is slowing down and as a result the moon is coming nearer to it

Reason (R): The angular momentum of the earth moon system is not conserved

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

47. Assertion (A): The length of the day is slowly increasing.

Reason (R): The dominant effect causing a slowdown in the rotation of the earth is the gravitational pull of other planets in the solar system.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

48. If the distance the earth and the sun were half its present value, the number of days in a year would have been

a) 129 b) 182.5 c) 730 d) 64.5

49. A satellite is rotating around a planet in the orbit of radius r with time period T. If gravitational force changes according to $r^{5/2}$, the T^2 will be

a) $\propto r^3$ b) $\propto r^{7/2}$

c) $\propto r^{9/2}$ d) $\propto r^{3/2}$

Key

1) **a**	2) **a**	3) **b**	4) **d**	5) **d**	6) **b**	7) **c**	8) **d**	9) **b**	10) **c**
11) **d**	12) **d**	13) **a**	14) **a**	15) **d**	16) **d**	17) **d**	18) **a**	19) **a**	20) **c**
21) **a**	22) **a**	23) **b**	24) **d**	25) **b**	26) **c**	27) **c**	28) **d**	29) **a**	30) **c**
31) **b**	32) **d**	33) **c**	34) **c**	35) **c**	36) **c**	37) **b**	38) **b**	39) **a**	40) **b**
41) **b**	42) **c**	43) **b**	44) **c**	45) **a**	46) **d**	47) **d**	48) **a**	49) **b**	

Hints

Newton's Law of Gravitation

1. From the law of conservation of angular momentum

$$mr_1v_1 = mr_2v_2$$

$$r_1v_1 = r_2v_2$$

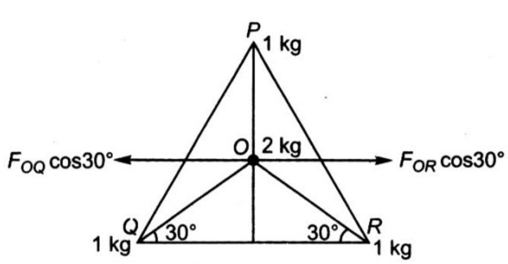
$$\frac{v_1}{v_2} = \frac{r_2}{r_1}$$

2. Concept

3. $g = \frac{GM_E}{R_E^2}$

Or $M_E = \frac{gR_E^2}{G}$

4. Given $OP = OQ = OR = \sqrt{2}m$



The gravitational force on mass 2kg due to mass 1kg at P

$$F_{OP} = G\frac{2\times 1}{(\sqrt{2})^2} = G \text{ along OP}$$

Similarly,

$$F_{OQ} = G\frac{2\times 1}{(\sqrt{2})^2} = G \text{ along OQ}$$

And $F_{OR} = G\frac{2\times 1}{(\sqrt{2})^2} = G$ along OR

$F_{OQ}\cos 30^0$ and $F_{OR}\cos 30^0$ are equal acting in opposite direction, then cancel out each other.

Then the resultant force on the mass 2kg at O

$$F = F_{OP} - (F_{OQ}\sin 30^0 + F_{OR}\sin 30^0)$$

$$F = G - \left(\frac{G}{2} + \frac{G}{2}\right)$$

F = 0

5. Concept

6. $$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g}{100} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \left(1+\frac{h}{R}\right)^2 = 100 \Rightarrow h = 9R$$

7. Acceleration due to gravity at earth's surface is given by

$$g = \frac{GM}{R^2} \text{ .... (i)}$$

Since earth is assumed to be spherical in shape, its mass is

$$M = volume \times density = \frac{4}{3}\pi R^3 \rho$$

Given, $\rho_e = \rho_p = \rho, G_p = 2G_e$

$$\therefore \frac{g_e}{g_p} = \frac{G_e\left(\frac{4}{3}\pi R_e^3\right)\rho \times R_p^2}{R_e^2 \times G_p\left(\frac{4}{3}\pi R_p^3\right)\rho}$$

$$1 = \frac{G_e R_e^3 \times R_p^2}{R_e^2 \times R_p^3 \times 2G_e} \qquad (\because G_p = 2G_e)$$

$$1 = \frac{R_e}{2R_p}$$

$$\Rightarrow \frac{R_p}{R_e} = \frac{1}{2}$$

8. Concept

9. $\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{1}{2}$

$\Rightarrow h = 2650km$

10. $\frac{mv^2}{R} = \frac{GMm}{(2R)^2}$

$\Rightarrow v = \frac{1}{2}\sqrt{\left(\frac{GM}{R}\right)}$

11. $F = \frac{Gm_1m_2}{r^2}$

And $F' = \frac{Gm_1m_2}{(3r)^2} = \frac{F}{9}$

$\therefore$ % decreases in $F = \left(\frac{F-F'}{F}\right)\times 100 = \frac{8}{9}\times 100$

Thus, attraction force between sun and earth is decreased by 89%.

12. Concept

13. Concept

Acceleration due to Gravity

14. Concept 15.

16. The acceleration due to gravity on an object of mass m

$$g = \frac{F}{m}$$

But from Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

Where M is the mass of the earth and R the radius of earth,

$$\therefore g = \frac{GMm/R^2}{m} = \frac{GM}{R^2}$$

Given, $\rho_{planet} = 2\rho_{earth}$

Also, $g_{planet} = g_{earth}$

$$\Rightarrow \frac{GM_p}{R_p^2} = \frac{GM_e}{R_e^2}$$

Or $\dfrac{G \times \frac{4}{3}\pi R_p^3 \rho_p}{R_p^2} = \dfrac{G \times \frac{4}{3}\pi R_e^2 \rho_e}{R_e^2}$

Or $R_p \rho_p = R_e \rho_e$

Or $R_p \times 2\rho_p = R_e \rho_e$

Or $R_p = \dfrac{R_e}{2} = \dfrac{R}{2}$

17. $g_A = 9g_B$ …………..(i)

But, $v^2 = 2gh$

At planet A, $h_A = \dfrac{v^2}{2g_A}$ …………..(ii)

At planet B, $h_B = \dfrac{v^2}{2g_B}$ …………….. (iii)

Dividing eq (i), $g_A = 9g_B$

$$\therefore \frac{h_A}{h_B} = \frac{g_B}{9g_B} = \frac{1}{9}$$

Or $h_B = 9h_A = 9 \times 2 = 18m$ ($\because h_A = 2m$)

18. $$g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = G \times \frac{4}{3}\pi R \rho$$

$$\Rightarrow g \propto R$$

$$\therefore \frac{g'}{g} = \frac{R'}{R}$$

$$\Rightarrow \frac{g'}{g} = \frac{3R}{R} = 3 \quad \Rightarrow g' = 3g$$

19. Concept

20. $$g' = g\left(1 - \frac{h}{R}\right)$$

$$\Rightarrow w' = w\left(1 - \frac{h}{R}\right)$$

$$h = \frac{R}{2}, w = 500N$$

$$\therefore w' = 500\left(1 - \frac{1}{2}\right) = \frac{500}{2} = 250N$$

Gravitational Energy

21. Energy required = Total energy (final) – total energy (initial)

$$= -\frac{GMm}{2(3R)}\left(-\frac{GMm}{2(2R)}\right) = \frac{GMm}{4R} - \frac{GMm}{6R} = \frac{GMm}{12R}$$

22. $v_e = \sqrt{\frac{2GM_e}{R_e}}$

$$M_p = \frac{M_e}{2}, R_p = \frac{R_e}{4}$$

$$v_e' = \sqrt{\frac{2G \times M_e \times 4}{2R_e}} = \sqrt{2}v_e$$

23. $U_e = -\frac{GMm}{R}$ and $U_h = -\frac{GMm}{(R+nR)} = -\frac{GMm}{R(n+1)}$

$$\Delta U = U_h - U_e = \frac{GMm}{R}\left\{1 - \frac{1}{(n+1)}\right\} = \left(\frac{n}{n+1}\right)\frac{GMm}{R} = \left(\frac{n}{n+1}\right)mgR$$

24. $U_i = -\frac{GMm}{r}$

$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$

$$U_i = -\frac{6.67 \times 10^{-11}}{0.1}$$

$$= -6.67 \times 10^{-10}\ J$$

$$\therefore W = \Delta U$$

$$= U_f - U_i \qquad (\because U_f = 0)$$

$$\therefore W = -U_i = 6.67 \times 10^{-10}\ J$$

25. By conservation of momentum

$$m_1 v_1 - m_2 v_2 = 0$$

$$\Rightarrow m_1 v_1 = m_2 v_2 \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

By conservation of energy

Change in PE = change in KE

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2Gm_1m_2}{r} \text{ ……………………. (ii)}$$

On solving eqs (i) and (ii)

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}} \text{ and } v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}$$

$$\therefore v_{app} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}(m_1+m_2)}$$

26. Applying law of conservation of energy for asteroid at a distance $10R_e$ and at earth's surface

$$K_i + U_i = K_f + U_f \text{ ……………….(i)}$$

Now, $K_i = \frac{1}{2}mv_i^2$ and $U_i = \frac{GM_em}{10R_e}$

$K_f = \frac{1}{2}mv_f^1$ and $U_f = \frac{GM_em}{R_e}$

Substituting these values in eqs (i), we get

$$\frac{1}{2}mv_i^2 - \frac{GM_em}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_em}{R_e}$$

$$\Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + \frac{GM_em}{R_e} - \frac{GM_em}{10R_e}$$

$$\therefore v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

27. Inside spherical shell gravitational field intensity at any point is zero, hence

$$I_P + I_Q = 0$$

$$\Rightarrow I_P + I_Q \neq 0$$

28. $v_{rms} = \sqrt{\frac{3kT}{m}}$ ………….. (i)

Escape velocity of gas molecules is

$v_{es} = \sqrt{2gR_e}$ ……….. (ii)

As the root mean square velocity of gas molecules must be equal to the escape velocity

$\therefore$ From eqs (i) and (ii) we get

$$\sqrt{\frac{3kT}{m}} = \sqrt{2gR_e} \Rightarrow T = \frac{2gR_e m}{3k}$$

$$\Rightarrow T = \frac{2\times 9.8\times 6.4\times 10^6 \times 0.34\times 10^{-26}}{3(1.38\times 10^{-23})} = 10^4 K$$

Therefore, $10^4 K$ is the temperature at which hydrogen molecules will escape form earth's surface.

29. Escape velocity $v_e = \sqrt{2gR}$

Where g is acceleration due to gravity and R is radius

$$\frac{R_1}{R_2} = K, \frac{g_1}{g_2} = g$$

$$\frac{v_1}{v_2} = \frac{\sqrt{g_1 R_1}}{\sqrt{g_2 R_2}} = \sqrt{Kg}$$

$$\frac{v_1}{v_2} = (Kg)^{1/2}$$

30. Escape velocity $v_e = \sqrt{\frac{2GM_e}{R_e}}$

$$M_e = M_p, R_p = \frac{R_e}{4}$$

$$\therefore \frac{v_p}{v_e} = \sqrt{\frac{M_e}{M_e} \times \frac{R_e}{R_{e/4}}} = \sqrt{4} = 2$$

$$\Rightarrow v_p = 2v_e = 2\times 11.2 = 22.4 kms^{-1}$$

31. Concept

Motion of Satellites and Kepler's Laws of Planetary Motion

32. Binding energy of satellite $= \frac{GMm}{2r}$

Where r is the radius of orbit

In second case $BE = \frac{GMm}{3r}$

$\therefore \Delta E = \frac{GMm}{r}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{GMm}{6r}$

% increase in energy of a satellite $= \frac{\frac{GMm}{6r} \times 100}{\frac{GMm}{2r}} = \frac{2}{6} \times 100 = 33.33\%$

33. $T^2 \propto R^3$ Or $T \propto R^{3/2}$

$\frac{T'}{T} = \left(\frac{R'}{R}\right)^{3/2}$

Or $\frac{T'}{T} = \left(\frac{4R}{R}\right)^{3/2} = (4)^{3/2} = (2^2)^{3/2} = 2^3 = 8$

$\therefore T' = 8T = 8 \times 90 = 720$ min

34. Concept

35. Concept

36. From second law,

$\frac{dA}{dt} = \frac{1}{2} r^2 \left(\frac{d\theta}{dt}\right) = \frac{1}{2} r^2 \omega$

Let J be angular momentum, I the moment of inertia and m the mass, then

$J = I\omega = mr^2\omega$

$\therefore \frac{dA}{dt} = \frac{J}{2m} =$ constant

Hence, angular momentum of the planet is conserved

37. Potential energy of satellite

$$U = -\frac{GM_e m}{R_e}$$

Where R_e is radius of earth, M_e the mass of earth, m the mass of satellite and G the gravitational constant.

$$|U| = \frac{GM_e m}{R_e}$$

Kinetic energy of satellite

$$K = \frac{1}{2}\frac{GM_e m}{R_e}$$

Thus, $\frac{K}{|U|} = \frac{1}{2}\frac{GM_e m}{R_e} \times \frac{R_e}{GM_e m} = \frac{1}{2}$

38. Minimum required energy $= U_2 - U_1 = \frac{-GMm}{2(3R)} - \left(\frac{-GMm}{R}\right)$

$$= \frac{GMm}{R}\left[-\frac{1}{6} + 1\right] = \frac{5}{6}\frac{GMm}{R} = \frac{5}{6}mgR$$

39. From Kepler's law, $T = kr^{3/2}$

$$\frac{dT}{dr} = \frac{3}{2}\frac{kr^2}{T}$$

$$\frac{dT}{dr} = \frac{3}{2}\left(\frac{T}{r}\right) \Rightarrow \frac{\Delta T}{\Delta r} = \frac{3}{2}\left(\frac{T}{r}\right)$$

$$\Delta T = \frac{3}{2}\left(\frac{T}{r}\right)\Delta r$$

40. Total energy of satellite,

$$E = U + K = \frac{GM_e m}{R_e} + \frac{1}{2}\frac{GM_e m}{R_e}$$

But, $\frac{mv^2}{R_e} = \frac{GM_e m}{R_e}$

Or $mv^2 = \frac{GM_e m}{R_e}$

$\therefore E = \frac{1}{2}mv^2$

41. Eccentricity $e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = 0.0167$

Therefore, $\frac{r_{max}}{r_{min}} = \frac{1+e}{1-e}$

$= \frac{1-0.0167}{1+0.0167}$

As angular momentum remains constant, hence

$r_{max} \times v_{min} = r_{min} \times v_{max}$

$\therefore \frac{v_{max}}{v_{min}} = \frac{r_{max}}{r_{min}} = \frac{1+0.0167}{1-0.0167}$ = 1.0339

42. Given $R_1 = R_{e'}$

$R_2 = R_e + \frac{R_e}{2} = \frac{3}{2}R_e$

But, $v_0 = \sqrt{\frac{GM_e}{R}} \Rightarrow v_0 \propto \sqrt{\frac{1}{R}}$

Or $\frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{3R_e}{2R_e}} = \sqrt{\frac{3}{2}}$

$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v_0 \ (\because v_1 = v_0)$

43. $T_1 =$ 1 year, $R_1 = R \ R_2 = 2R$

But, $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{R}{2R}\right)^3 = \frac{1}{8}$

$\Rightarrow T_2 = 2\sqrt{2}T_1 = 2\sqrt{2}years$

44. $F_1 = \frac{GMm}{r_1^2} = \frac{gR^2m}{r_1^2}$

Where $GM = gR^2$

Similarly $F_2 = \frac{GMm}{r_2^2} = \frac{gR^2m}{r_2^2}$

This gravitational force provides the necessary centripetal force, hence we have

$\frac{gR^2m}{r_1^2} = \frac{mv_1^2}{r_1}$

$\Rightarrow v_1^2 = \frac{gR^2}{r_1}$

Similarly $v_2 = \sqrt{\frac{gR^2}{r_2}}$

$\therefore \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$

Given $r_1 > r_2$

$\therefore v_2 > v_1$

45. Concept

46. Concept

47. Concept

48. From Kepler's third law

$\left(\frac{T}{365}\right)^2 = \left(\frac{1}{2}\right)^2$

$\therefore T = \frac{365}{(2)^{3/2}} = 129 days$

49. Gravitational force = centripetal force

$\frac{GMm}{r^{5/2}} = \frac{mv^2}{r}$ or $v^2 = \frac{GM}{r^{3/2}}$

Now $T^2 = \left(\frac{2\pi r}{v}\right)^2 = \frac{4\pi^2 r^2}{GM / r^{3/2}} = \frac{4\pi^2 r^{7/2}}{GM}$ or $T^2 \propto r^{7/2}$