Dynamics

Work–Power–Energy

1. Conservative force

a) A force is said to be conservative if the work done by it is independent of path followed by the body.

b) Work done by a conservative force for a closed path is zero.

c) Work done by a conservative force depends only on the initial and final positions of the body.

d) Work done by a conservative force is the product of Force and displacement.

e) During a round trip the body attains the same initial K.E.

Ex. Gravitational force, Electrostatic force etc.

2. Non - Conservative force

a) A force is said to be non- conservative if the work done by it depends on the path followed by the body.

b) Work done by a non-conservative force for a closed path is not zero.

c) During a round trip the body attains a different K.E. as that of initial.

d) Work done by a non-conservative force is the product of Force and distance.

e) Due to a non-conservative there may be a loss of mechanical energy but the total energy is constant.

Ex. Frictional force

3. Work

a. Work is said to be done when the point of application of force has some displacement in the direction of the force.

b) The amount of work done is given by the dot product of force and displacement.

$$W = \vec{F}.\vec{s} = Fs\cos\theta$$

c) Work is independent of the time taken and is a scalar.

d) If the force and displacement are perpendicular to each other, then the work done is zero.

e) A person rowing a boat upstream is at rest with respect to an observer on the shore. According to the observer the person does not perform any work. However, the person performs work against the flow of water. If he stops rowing the boat, the boat moves in the direction of flow of water and work is performed by the force due to flow, as there is displacement in the direction of flow.

f) If the work is done by a uniformly varying force such as restoring force in a spring, then the work done is equal to the product of average force and displacement.

g) If the force is varying non–uniformly, then the work done = $\int \vec{F}.\vec{ds} = \int F.ds.\cos\theta$.

h) The area of F–s graph gives the work done.

i) SI unit of work is joule.

j) Joule is the work done when a force of one Newton displaces a body through one metre in the direction of force.

k) CGS unit of work is erg; 1 J = 10^7 ergs.

l) If the force or its component is in a direction opposite to the displacement, the work is negative.

Ex. When a body is lifted vertically upwards, the work done by the gravitational force is negative, as the displacement is upward whereas the gravitational force is acting downwards.

m) The work done in lifting an object of mass m through a height 'h' is equal to mgh.

n) When a body of mass m is raised from a height h_1 to height h_2, then the work done = mg $(h_2 - h_1)$.

o) Let a body be lifted through a height 'h' vertically upwards by a force 'F' acting upwards. Then, the work done by the resultant force is W = (F – mg)h.

p) The work done on a spring in stretching or compressing it through a distance x is given $W = \frac{1}{2}kx^2$ where k is the force constant or spring constant.

q) Work done in changing the elongation of a spring from x_1 to x_2 is $W = \frac{1}{2}k(x_2^2 - x_1^2)$.

r) The work done in pulling the bob of a simple pendulum of length L through an angle

$W = mgL(1 - \cos\theta)$

s) The work done in lifting a homogeneous metal rod lying on the ground such that it makes an angle 'θ' with the horizontal, is $W = \frac{mgl\sin\theta}{2}$.

t) The work done in rotating a rod or bar of mass m through an angle θ about a point of suspension is $W = \frac{mgL}{2}(1-\cos\theta) = mgL\sin^2(\theta/2)$ where L is the distance of the centre of gravity from the point of suspension.

u) The work done in lifting a body of mass 'm' and density 'd_s' in a liquid of density 'd_l' through a height 'h' under gravity is $W = m\,g\,h\left(1-\frac{d_l}{d_s}\right)$

v) Work done in pulling back a $\frac{1}{n}^{th}$ part of length of a chain hanging from the edge onto a smooth horizontal table completely is $W = \frac{mgl}{2n^2}$.

w) Inclined plane

i. Work done in moving a block of mass 'm' up a smooth inclined plane of inclination 'θ' through a distance 's' is $W = Fs = (mg\sin\theta)\,s$.

ii. If the plane is rough, then $W = mg(\sin\theta+\mu_k\cos\theta)s$

x) Work done by a position dependent force

If the position of a body changes from x_1 to x_2 then work done is given by

$W=\int_{x_1}^{x_2} F\,dx$ = area under F – S curve.

y) Work done by a time dependent force

$$a = \frac{F}{m}$$

Now $a=\frac{dv}{dt} \Rightarrow v=\int dv = \int a\,dt$

And $W = \frac{1}{2}m\left(v^2-u^2\right)$

z) Work done when position depends on time

$$v = \frac{ds}{dt}$$

When t= t_1 v= v_1 and when t= t_2 v= $v_{2,\text{ then}}$

$$W = \frac{1}{2}m\left(v_2^2-v_1^2\right).$$

4. Springs

a) The restoring force on the spring per unit elongation is called force constant or spring constant F = - Kx

(Negative sign indicates that the force is opposite to elongation)

b) The work done in stretching or compressing a string through x is given by $W = \frac{1}{2}Kx^2$

c) $W = \frac{F}{2x} \times x^2 = \frac{1}{2}Fx \qquad \left(\because K = \frac{F}{x}\right)$

d) $W = \frac{1}{2} \times F \times \frac{F}{K} = \frac{F^2}{2k} \qquad \left(\because x = \frac{F}{k}\right)$

e) If the spring is stretched from x_1 and x_2 then the work don is given by $W = \frac{1}{2}K\left(x_2^2 - x_1^2\right)$

Also, $W = \frac{1}{2}(F_2 x_2 - F_1 x_1)$

f) If a body is dropped from a height h on to a spring of constant k, and if x is the compression in the spring, then $mg(h+x) = \frac{1}{2}kx^2$

g) If air friction is considered, $(mg - f)(h+x) = \frac{1}{2}kx^2$

h) If a body of mass on moving with a speed v collides a spring in its pith and compresses the spring through 'x' then, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$. If friction is considered.

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 - f.s = \frac{1}{2}mv^2 - \mu mgs$

i) If a spring of spring constant 'k' is cut into n equal parts the spring constant of each part is 'nk'

j) If a spring of constant k is cut into unequal parts, then. $k_1 l_1 = k_2 l_2 = k_3 l_3 = ... = k\left(l_1 + l_2 + l_3 ...\right)$

5. Power

a. Rate of doing work is called power.

Power = $\frac{\text{work}}{\text{time}}$ = Force x velocity.

b. SI unit of power is watt and CGS unit is erg/second.1 horse power = 746 watt.

c. If a vehicle travels with a speed of v overcoming a total resistance of F, then the power of the engine is given by $P = \vec{F}.\vec{v}$.

d. If a body is rotated in circular path, the power exerted is given by $P = \tau\frac{d\theta}{dt} = \tau\omega$

e. If a block of mass 'm' is pulled along the smooth inclined plane of angle 'θ', with constant velocity 'v', then the power exerted is, $p = (mg \sin\theta)v$

f. If the block is pulled up a rough inclined plane then the power is $P = mg(\sin\theta + \mu_k \cos\theta)v$.

g. If the block is pulled down a rough inclined plane then the power is $P = mg(\sin\theta - \mu_k \cos\theta)v$.

h. When water is coming out from a hose pipe of area of cross section 'A' with a velocity 'v' and hits a wall normally and

i) stops dead, then force exerted by the water on the wall is $Av^2\rho$. And the power exerted by water is $P = Av^3\rho$ (ρ = density of water)

ii) If water rebounds with same velocity (v) after striking the wall, $P = 2Av^3\rho$

i. When sand drops from a stationary dropper at a rate of $\frac{dm}{dt}$ on to a conveyer belt moving with a constant velocity, then the extra force required to keep the belt moving with a constant speed V is given by $F = v.\frac{dm}{dt}$ and the power required $= P = \frac{dm}{dt}v^2$.

j. If a pump lifts the water from a well of depth 'h' and imparts some velocity 'v' to the water, then the power of pump $P = \frac{mgh + \frac{1}{2}mv^2}{t}$

k. Power exerted by a machine gun which fires 'n' bullets in time 't' is $P = \frac{mnv^2}{2t}$

l. If a pump delivers V litres of water over a height of h metres in one minute, then the power of the engine $(P) = \frac{Vgh}{60}$.

m. A motor sends a liquid with a velocity 'V' in a tube of cross section 'A' and 'd' is the density of the liquid, then the power of the motor is $P = \frac{1}{2}AdV^3$

n. A body of mass M initially at rest on a smooth horizontal surface accelerates uniformly and acquires velocity V_1 in time t_1. The work done on the body in time t is

i) Work done $= \frac{mv_1^2}{2t_1^2}t^2$

ii) Average power $= \frac{W}{t} \quad = \frac{1}{2}\frac{mv_1^2}{t_1^2}t$

iii) Instantaneous power = Fv$= \frac{mv_1^2}{t_1^2}t$

6. Energy

a) The energy possessed by a body by virtue of its motion is called kinetic energy. Ex. A fired bullet, blowing wind, etc

For a body of mass m is moving with a velocity v, kinetic energy $= \frac{1}{2}mv^2$.

b) The energy possessed by a body by virtue of its motion is called kinetic energy.

Ex. A bent bow, Water stored in a reservoir etc.

c) A flying bird possesses both K.E. and P.E.

d) The work done in lifting an object of mass m through a height 'h' is stored as potential energy in the body and it is equal to mgh.

e) **Work–energy theorem:** The work done by the constant resultant force acting on a body is equal to the change in its kinetic energy. $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

f) If the kinetic energy of a body of mass m is E and its momentum is P, then $E = \frac{P^2}{2m}$.

g) If the momentum of the body increased by 'n' times, K.E increase by n^2 times.

h) If the K.E of the body increases by 'n' times, the momentum increases by $\sqrt{n}$ times.

i) If the momentum of the body increases by P%, % increase in K.E.$=\left(2+\frac{P}{100}\right)$ P%

j) If the momentum of the body decreases by p%, % decrease in K.E. = $\left(2-\frac{P}{100}\right)$ p%.

k) If the K.E of the body increases by E%, % increase in momentum$=\left(\sqrt{1+\frac{E}{100}}-1\right)100\%$.

l) If the K.E of the body decreases by E%, % decrease in momentum$=\left(1-\sqrt{1+\frac{E}{100}}\right)100\%$.

m) If two bodies, one heavier and the other lighter are moving with the same momentum, then the lighter body possesses greater kinetic energy.

n) If two bodies, one heavier and the other lighter have the same K.E. then the heavier body possesses greater momentum.

o) Two bodies, one is heavier and the other is lighter are moving with the same momentum. If they are stopped by the same retarding force, then

i) The distance travelled by the lighter body is greater. ($s \propto \frac{1}{m}$)

ii) They will come to rest within the same time interval

p) Two bodies, one is heavier and the other is lighter are moving with same kinetic energy. If they are stopped by the same retarding force, then

i) The distance travelled by both the bodies is same.

ii) The time taken by the heavier body will be more. ($t \propto \sqrt{m}$)