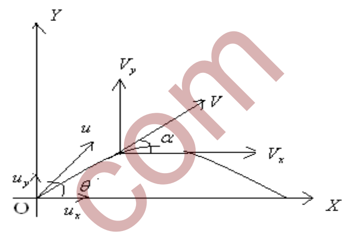
Motion in a Plane Oblique Projection



1. A body which has uniform velocity in the horizontal direction and uniform acceleration in the vertical direction is called a projectile.

2. The path of a projectile is called trajectory and it is a parabola.

3. For a projectile, the horizontal component of velocity ($u_x = u\cos\theta$) remains constant throughout its motion.

4. The vertical component ($u_y = u\sin\theta$) is subjected to acceleration due to gravity.

5. **Equations of a projectile**

a) Maximum height reached = $\frac{u^2\sin^2\theta}{2g}$

b) Time of flight = $\frac{2u\sin\theta}{g}$

Time of ascent = time of descent = $\frac{u\sin\theta}{g}$

c) Range = $\frac{u^2\sin 2\theta}{g}$

d) $\tan\theta = \frac{4H_{max}}{R}$ and $\tan\theta = \frac{gT^2}{2R}$

6. At the highest point of the projectile

a) Velocity is $u\cos\theta$ (minimum).

b) Vertical component of velocity is zero.

c) KE of the body is $\frac{1}{2}mu^2\cos^2\theta$.

d) PE of the body is $\frac{1}{2}mu^2\sin^2\theta$.

e) Angle between the velocity and acceleration is 90°.

f) The direction of motion of the body is horizontal.

7. Velocity after time "t"

a) Vertical component of velocity $V_y = u \sin\theta - gt$

b) Velocity of a projectile after t seconds $v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$.

c) The angle made by a projectile after t seconds, then $\tan\alpha = \dfrac{u\sin\theta - gt}{u\cos\theta}$.

8. If projected from level ground

a) Velocity of the projectile when it moves perpendicular to its initial velocity is U cotθ.

b) Time taken for the velocity to become perpendicular to the initial velocity is $\dfrac{u}{g\sin\theta}$

9. Position of the projectile after time 't'

a) If x and y represent the horizontal and vertical displacements with respect the point of projection't' seconds after projection

$x = (u\cos\theta)\, t$

$y = (u\sin\theta)\, t - \dfrac{1}{2} g t^2$

b) Equation of trajectory is

$$Y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

10. If $y = Ax - Bx^2$, then

a) The angle of projection $\theta = \tan^{-1} A$

b) Maximum height $H = \dfrac{A^2}{4B}$

c) Range $R = \dfrac{A}{B}$

d) Time of flight $T = \sqrt{\dfrac{2}{Bg}}$

e) Velocity of projection $u = \sqrt{\dfrac{g(A^2+1)}{2B}}$

11. At the half of the maximum height

a) Vertical component of velocity is $\frac{u\sin\theta}{\sqrt{2}}$

b) Horizontal component of velocity is u cosθ

c) Velocity of the body is $u\left(\frac{1+\cos^2\theta}{2}\right)^{1/2}$

12. If the angle of projection is θ and (90 - θ) (Complementary angles)

a) Range is same

b) Sum of maximum heights is $\frac{u^2}{2g}$

c) Ratio of max heights is $\tan^2\theta:1$

d) Ratio of times of flight is tanθ : 1

e) If h_1 and h_2 are the maximum heights, then $R = 4\sqrt{h_1 h_2}$

f) Range = $\frac{1}{2}gT_1T_2$ where T_1 and T_2 are the times of flights.

13. If a man throws a body to a maximum distance R then he can project the body to vertical height R/2.

14. If a body is projected down at an angle θ with the horizontal from the top of a tower then

$$h = -(u\sin\theta)\,t + \frac{1}{2}gt^2$$