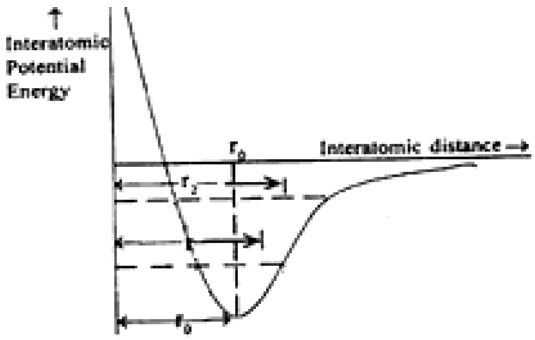
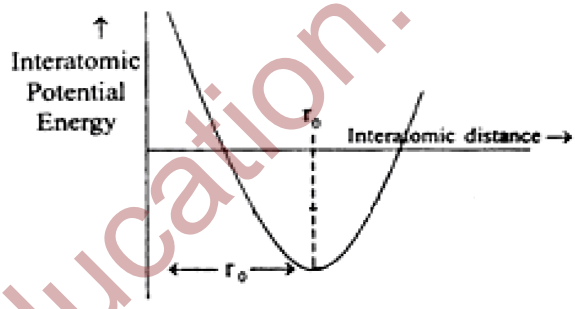
Expansion of Solids

1. Lattice vibrations are purely harmonic and then PE curve is a symmetric parabola indicating no thermal expansion the average inter-atomic distance remains same
2. If the lattice vibrations are an-harmonic, PE curve is not a symmetric parabola indicating the thermal expansion. The average inter-atomic distance increases with increase of temperature.





3. Coefficient of linear expansion (α): The ratio of increase in length per one degree rise in temperature to its original length is called coefficient of linear expansion. $\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}$

   Unit of α : $C^{o^{-1}}$ or K^{-1}

4. Coefficient of area or superficial expansion (β): The increase in area per unit area per one degree rise in temperature is called coefficient of areal expansion. $\beta = \frac{a_2 - a_1}{a_1(t_2 - t_1)}$

   Unit of β : $C^{o^{-1}}$ or K^{-1}

5. The coefficient of volume or cubical expansion (γ) is the increase in volume per unit volume per degree rise in temperature. $\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)}$

   Unit of γ: $C^{o^{-1}}$ or K^{-1}

6. If α_x, α_y and α_z denotes coefficient of linear expansions along X, Y, Z directions respectively, then $\alpha_{avg} = \frac{\alpha_x + \alpha_y + \alpha_z}{3} \Rightarrow \gamma = \alpha_x + \alpha_y + \alpha_z$

7. $\alpha : \beta : \gamma = 1:2:3$ or $\gamma = 3\alpha$; $\beta = 2\alpha$;

$\gamma = \alpha + \beta$.

8. The numerical value of coefficient of linear expansion of a solid depends on the nature of the material and the scale of temperature used.

9. The numerical value of coefficient of linear expansion of a solid is independent of physical dimensions of the body and also on the unit of length chosen.

10. The numerical value of α or β or γ in the units of per °C is 9/5 times its numerical value in the units of per °F.

α per °F $= \frac{5}{9}.\alpha$ per °C.

α per °R $= \frac{5}{4}.\alpha$ per °C.

. α per K = α in /°C

11. The density of solid decreases with increase of temperature. $d_t = \frac{d_o}{1+\gamma t}$ Where d_o is density at 0°C.

12. If two rods of different materials have the same difference between their lengths at all temperatures, then $\alpha_1 L_1 = \alpha_2 L_2$

If the constant difference in their lengths is x, then $L_1 = \left(\frac{\alpha_2}{\alpha_1 \sim \alpha_2}\right)x$ and $L_2 = \left(\frac{\alpha_1}{\alpha_1 \sim \alpha_2}\right)x$

13. Pendulum clocks lose or gain time as the length increases or decreases respectively.

The fractional change= $\frac{\Delta T}{T} = \frac{\alpha \Delta t}{2}$.

The loss or gain per day= $\frac{\alpha \Delta t}{2}$ x86400 seconds.

14. If a metal rod is prevented from expansion on heating, stress will be developed, which is known as thermal stress.

Thermal stress developed in the rod, $\frac{F}{A} = Y\alpha\Delta t$

Thermal stress is independent of length of the rod.

15. As temperature increases, moment of inertia of a body increases. The Fractional change in moment of inertia is $\frac{\Delta I}{I} = 2\alpha t$

16. When the scale & body both are expanding $l_{correct} = l_{scale}\left[1 \pm \left(\alpha_{body} - \alpha_{scale}\right)t\right]$

17. A metal scale is calibrated at a particular temperature does not give the correct measurement at any other temperature.

a) When scale expands correction to be made $\Delta\lambda=\Lambda\ \alpha\ \Delta t$, correct reading=L+Δl

b) When scale contracts correction to be made $\Delta l=L\ \alpha\ \Delta t$, correct reading=$\tilde{L}\Delta l$. L=measured value.

$L_{measured}=L_{true}[1-\alpha(\Delta t)]$

18. For a mercury barometer $H_{correct}=H_{scale}\left[1-\left(\gamma_{Hg}-\alpha_{scale}\right)t\right]$

19. Applications of expansions of solids

a. A cavity of a solid object expands on heating just like a solid object of the same volume.

b. If a thin rod and a thick rod of same length and material are heated to same rise in temperature, both expand equally.

c. If a thin rod and a thick rod of same length and material are heated by equal quantities of heat, thin rod expands more than thick rod.

d. A metal plate contains two holes at a certain distance apart from each other. If the plate is heated, the distance between the centers of the holes increases.

e. Platinum is used to seal inside glass because both have nearly equal coefficients of linear expansion.

f. Iron or steel is used for reinforcement in concrete because both have nearly equal coefficients of expansion.

g. Pyrex glass has low α. Hence combustion tubes and test tubes for hating purpose are made out of it.

h. Invar steel (steel + nickel) has very low α. So it is used in making pendulum clocks, balancing wheels and measuring tapes. (Composition of invar steel is 64% steel and 36% nickel).

i. Thick glass tumbler cracks when hot liquid is poured into it because of unequal expansion.

j. Hot chimney cracks when a drop of water falls on it because of unequal contraction.

k. A brass disc snuggly fits in a hole in a steel plate. To loosen the disc from the hole, the system should be cooled.

l. To remove a tight metal cap of a glass bottle, it should be warmed.

m. While laying railway tracks, small gaps are left between adjacent rails to allow for free expansion without affecting the track during summer. Gap to be left $(\Delta l)=\alpha l \Delta t$=expansion of each rail.

n. Concrete roads are laid in sections and expansion channels are provided between them.

o. Thermostat is a device which maintains a steady temperature.

p. Thermostats are used in refrigerators, automatic irons and incubators.

q. Thermostat is a bimetallic strip made of iron and brass. The principle involved are different materials will have different coefficients of linear expansion.

$= Y \Delta \alpha \Delta t = 2 \times 10^{11} \times 11 \times 10^{-6} \times 100 = 2.2 \times 10^{8}$ N/m^2