Product of Vectors

1. Dot Product

a) Scalar product (or) dot product is defined as the product of the magnitudes of two vectors and the cosine of the angle between them. The dot product of two vectors $\vec{a}$ and $\vec{b}$ is given by $\bar{a}.\bar{b} = ab\cos\theta$

b) Scalar product is commutative i.e., $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

c) Scalar product is distributive i.e., $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

d) If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $\vec{a}.\vec{b}$ = ab.

e) If $\bar{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}.\vec{b} = 0$.

f) If $\bar{a}$ and $\vec{b}$ are anti-parallel vectors, then $\vec{a}.\vec{b}$ = –ab.

g) Component of $\bar{a}$ along $\bar{b}$ $a\cos\theta = \dfrac{\bar{a}.\bar{b}}{|\bar{b}|}$

h) Component of $\bar{b}$ along $\bar{a}$ $(b\cos\theta) = \dfrac{\bar{a}.\bar{b}}{|\bar{a}|}$

i) Vector component of $\bar{a}$ along $\bar{b} = a\cos\theta \times \dfrac{\bar{b}}{|\bar{b}|}$

j) Vector component of $\bar{b}$ along $a = b\cos\theta \times \dfrac{\bar{a}}{|\bar{a}|}$

k) In the case of unit vectors,

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$.

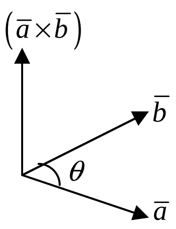
l) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

2. Applications of Dot Product

a) The dot product of force and displacement is called work done $W = \vec{F}.\vec{S}$.

b) The dot product of force and velocity is called power $P = \vec{F}.\vec{V}$.

3. Cross Product



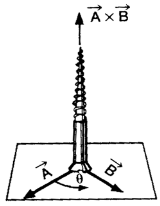
a) Cross product (or) vector product of two vectors is a vector which is the product of the magnitudes of the two vectors and the sine of the angle between them.

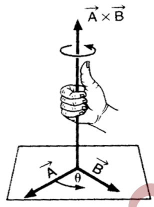
$\bar{A}\times\bar{B} = AB\sin\theta.\ \hat{n}$

Where $\hat{n}$ is a u nit vector along $\overline{A}\times\overline{B}$.

Eg: angular momentum ($\vec{L}=\vec{r}\times\vec{\omega}$), torque ($\vec{\tau}=\vec{r}\times\vec{F}$), angular velocity ($\vec{V}=\vec{\omega}\times\vec{r}$) etc.

b) The direction of $\overline{A}\times\overline{B}$ can be known from right hand thumb rule (or) Cork screw rule.





c) $i\times i=j\times j=k\times k=0$

$i\times j=k$ $\qquad j\times i=-k$

$j\times k=i$ $\qquad k\times j=-i$

$k\times i=j$ $\qquad i\times k=-j$

d) If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=$

$(A_yB_z-A_zB_y)\hat{i}-(A_xB_z-A_zB_x)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$

e) If $\vec{A}\times\vec{B}=0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then they are parallel to each other.

$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$;

$m(\vec{A}\times\vec{B})=(m\vec{A})\times\vec{B}=\vec{A}\times(m\vec{B})$

f) $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ (Anti-commutative)

g) The area of the triangle formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\frac{1}{2}\left|\vec{A}\times\vec{B}\right|$.

h) The area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\left|\vec{A}\times\vec{B}\right|$.

i) If $\vec{A}$ and $\vec{B}$ are diagonals of a parallelogram, then area of parallelogram= $\frac{1}{2}\left|(\overline{A}\times\overline{B})\right|$.

4. Applications of Cross Product

a) Torque is the cross product of radius vector and force vector, $\vec{\tau}=\vec{r}\times\vec{F}$

b) Angular momentum is the cross product of radius vector and linear momentum, $\vec{L}=\vec{r}\times\vec{p}$