Rotatory Motion

Horizontal Circular Motion

1. In translatory motion, every point in the body follows the path of its preceding one with same velocity including the centre of mass.
2. In rotatory motion, every point move with different velocity with respect to the axis of rotation. The particle on the axis of rotation will have zero velocity.
3. The angle described by the radius vector in a given interval of time is called the angular displacement.
4. Angular displacement is a vector passing through the centre and directed along the perpendicular to the plane of the circle whose direction is determined by right hand screw rule (It is a pseudo vector).
5. Angular displacement is measured in radians.
6. The rate of change of angular displacement is called angular velocity (ω).

   $$\omega = \frac{\theta}{t}\, rads^{-1}\ .$$

7. Angular velocity is a vector lying in the direction of angular displacement.
8. Linear velocity $(\vec{V}) = \vec{\omega} \times \vec{r}$.
9. Rate of change of angular velocity is called angular acceleration (α). Unit is rads^{2}.

   $$\alpha = \frac{\text{change in angular velocity}}{\text{time}}\ .$$

10. Linear acceleration = radius × angular acceleration. $\vec{a} = \vec{\alpha} \times \vec{r}$.
11. Resultant acceleration a = ω where a_r = radial acceleration and a_T = tangential acceleration.
12. Comparison of linear and angular quantities.

Translatory motion	Rotatory motion

v = u + at	$\omega_2 = \omega_1 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$
$s = \frac{u+v}{2}$	$\theta = \frac{\omega_1 + \omega_2}{2}$
Mass (m)	Moment of inertia (I)
F = ma	Torque $\tau = I\alpha$
Impulse = Ft	Angular impulse = $\tau\ t$
Linear momentum p = mv	Angular momentum L = I ω
Work = FS	$W = \tau\theta$
Power = FV	$P = \tau\omega$
$m_1v_1 = m_2v_2$	$I_1\omega_1 = I_2\omega_2$
$E = \frac{p^2}{2m}$	$E = \frac{L^2}{2I}$
$KE = \frac{1}{2}mv^2$	Rotational KE = $\frac{1}{2}I\omega^2$

13. If a particle makes n rotations per second $\theta = 2\pi n$.

14. Angular velocity is a pseudo vector (or) axial vector. $\bar{v} = \bar{\omega} \times \bar{r}$ and $\bar{v}.\bar{r} = \bar{r}.\bar{\omega} = 0$.

15. Rate of change of angular velocity is called angular acceleration (α). Unit is rad s^{-2}.

$\alpha = \frac{\text{change in angular velocity}}{\text{time}}$ The angular acceleration is a pseudo (or) axial vector.

16. The direction angular acceleration (α) is along the change in angular velocity.

$\bar{a} = \bar{\alpha} \times \bar{r}$ and $\bar{a}.\bar{r} = \bar{a}.\bar{\alpha} = 0$.

17. The direction α will be same as that of ω if it is increasing and opposite to that of ω if it is decreasing.

18. Uniform circular motion

a. Tangential acceleration is due to change in the speed and normal acceleration is due to the change in the direction.

b. Tangential acceleration $a_T = \frac{dv}{dt} = r\alpha$. This is along the tangent drawn along the circular path.

c. For vertical circular motion the tangential acceleration is given by $a_T = r\alpha = g\sin\theta$

d. Radial (or) normal (or) centripetal acceleration $a_N = \frac{v^2}{r} = r\omega^2 = 4\pi^2 n^2 r$

e. In uniform circular motion: (ω=constant)

i) Tangential acceleration is zero ($a_t = 0$)

ii) Normal acceleration a_N = constant

19. In non – uniform circular motion

Net acceleration $a = \sqrt{a_N^2 + a_T^2}$

(or) $a = \sqrt{\left(\frac{v^2}{r}\right)^2 + (r\alpha)^2}$ and $a = a_N i + a_T j$

20. $a_N = 0$ and $a_T = 0$ – uniform linear motion.

21. $a_N = 0$ and $a_T \neq 0$ - accelerated (or) non uniform linear motion.

22. $a_N \neq 0$ and $a_T = 0$ – uniform circular motion.

23. $a_N \neq 0$ and $a_T \neq 0$ - non uniform circular motion.

24. The force which makes a body move round a circular path with uniform speed is called the centripetal force. This is always directed towards the centre of the circle.

Centripetal force$=\dfrac{mv^2}{r} = mr\omega^2 = 4\pi^2n^2mr$.

25. A body moving round a circular path with uniform speed experiences an inertial or pseudo force which tends to make it go away from the centre. This force is called the centrifugal force and this is due to the inertia of the body.

26. Centrifugal force $\simeq$ centripetal force (but these are not action and reaction).

27. No work is done by centripetal force.

28. The kinetic energy of the body revolving round in a circular path with uniform speed is 'E'. If 'F' is the required centripetal force, then $F = \dfrac{2E}{r}$

29. Uses of centrifugal forces and centrifugal machines.

i) Cream is separated from milk (cream separator)

ii) Sugar crystals are separated from molasses.

iii) Precipitate is separated from solution.

iv) Steam is regulated by Watt's governer.

v) Water is pumped from a well (Electrical pump).

vi) Hematocentrifuge, Grinder, Washing machine, etc.

30. The angle through which a cyclist should lean while taking sharp turnings is given by the relation $\theta = \mathrm{Tan}^{-1}\left(\dfrac{v^2}{rg}\right)$.

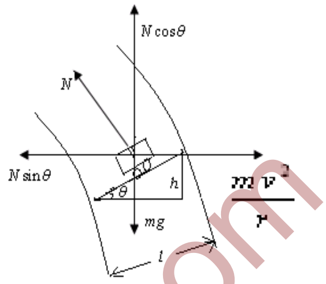
31. Safe speed on an unbanked road when a vehicle takes a turn of radius r is v $=\sqrt{\mu rg}$ where μ = coefficient of friction.

32. The maximum speed that is possible on curved unbanked track is given by g = v^2h/ar

Where h = height of centre of gravity and a = half the distance between wheels.

33. Angle of banking

At curves the outer edge of the road is slightly above the lower edge. The angle made by the tilled road with the horizontal is called angle of banking.



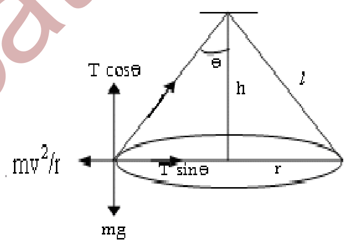
$N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg$

$\therefore Tan\,\theta = \frac{v^2}{r\,g}$

For small angles,

$\tan\theta = \sin\theta \quad \Rightarrow \frac{h}{l} = \frac{v^2}{r\,g} \Rightarrow h = \frac{v^2 l}{r\,g}$

34. Conical Pendulum: Let Tbe the tension in the string.



a. $T\sin\theta = \frac{mv^2}{r}$ And $T\cos\theta = mg$

$Tan\;\theta = \frac{v^2}{rg} = \frac{r\omega^2}{g}$

b. $Tan\;\theta = \frac{r}{h} \;\Rightarrow \frac{r}{h} = \frac{r\omega^2}{g} \Rightarrow \omega = \sqrt{\frac{g}{h}}$

c. Time period $T' = 2\pi\sqrt{\frac{h}{g}}$ (Or) $T' = 2\pi\sqrt{\frac{l^2 - r^2}{g}}$

And frequency $n = \frac{1}{2\pi}\sqrt{\frac{g}{h}}$.

d. $T\sin\theta = mr\omega^2 = m\left(l\sin\theta\right)\omega^2$.