Interference

1. Wave Front: Wave front is defined as the locus of the particles of the medium which are in the same state of vibration.

The line along which energy is propagated is called **a ray**. It is perpendicular to wave front at any point.

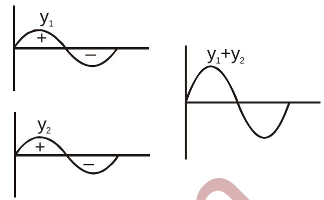
Types of wave front

- **Spherical wave front:** A wave front due to a point source in a three dimensional isotropic space is called spherical wave front. In case of spherical wave front intensity varies as $1/r^2$.
- **Cylindrical wave front:** If the source is a linear source of light, the wave front is a cylindrical wave front. In case of cylindrical wave front intensity varies as 1/r.
- **Plane wave front:** When the source is at infinite distance, the radius of curvature of wave front will be very large. A small portion of spherical wave front with source at infinite distance is a plane wave front. In case of plane wave front intensity is constant.

1. **Huygens's principle:** Each point on the wave front becomes a source of secondary disturbance and sends secondary wavelets which travels with the same speed as that of the original waves.
2. A surface tangential to all these secondary wavelets is the new wave front. Huygens's theory explained satisfactorily phenomena like reflection, refraction, interference and diffraction of light.
3. Secondary wavelets spread out as spherical secondary wave fronts with the speed of light.
4. The tangential surface to all the secondary wave fronts gives the new wave front.
5. The intensity of the secondary wave front is given by $I=I_0\,(1+\cos\phi)$ where ϕ is the angle between the original direction of propagation and the direction of observation. This shows that the secondary wave front has zero intensity in the backward direction.

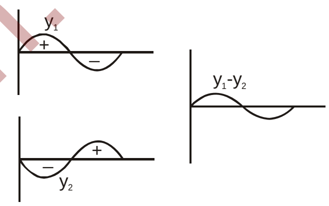
Principle of superposition of light waves

- When two or more than two waves superimpose over each other at a common point of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves i.e., $\vec{y} = \vec{y}_1 + \vec{y}_2$.



- When the displacement due to two wave are in the same direction (i.e., same phase) then resultant displacement is $y = y_1 + y_2$.

- When the displacement due to two waves are mutually in opposite direction (i.e., opposite phase) then resultant displacement will be
  $y = y_1 - y_2$.



If two waves are represented by $y_1 = a_1 \sin(\omega t)$ and $y_2 = a_2 \sin(\omega t + \phi)$, the resultant wave equation is $y = y_1 + y_2 = A \sin(\omega t + \theta)$ where the resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$ and phase angle

$$\theta = \mathrm{Tan}^{-1}\left(\frac{a_2\sin\phi}{a_1 + a_2\cos\phi}\right)$$

The resultant wave is also a harmonic wave of the same frequency. The resultant wave amplitude A will be maximum when $\cos\phi = 1$ i.e., when $\phi = 2n\pi$, n=0, 1, 2…

$A_{max} = a_1 + a_2$

The interference is then said to be **constructive**.

The resultant wave amplitude will be minimum when $\cos\phi = -1$, i.e., when $\phi = (2n+1)\pi$, n=0, 1, 2,..

$A_{min} = |a_1 - a_2|$

The interference is then said to be **destructive**.

Non coherent and coherent sources of light

1. Non coherent sources – Two sources of light, whose frequencies are not same and phase difference between the waves emitted by which does not remain constant with respect to time, are defined as non coherent sources.

2. The light emitted by two independent sources (candles, bulbs etc.) is non-coherent and interference phenomenon cannot be produced by such two sources.

3. The intensity of light or positions of fringes keep or changing in 10^{-8} second.

4. **Coherent sources** – The two sources of light, whose frequencies (or wavelength λ) are same and the phase difference between the waves emitted by which remains constant with respect to time are defined as coherent sources.

   They are obtained from the same single source.

   These can be apparent or real.

   Laser light is highly coherent and monochromatic.

   The distance between them is small.

   Their state of polarization is the same.

   These sources are of two types (i) Spatial coherent sources, (ii) Temporal coherent sources.

   There are two methods of obtaining these sources (i) Division of wave front, (ii) Division of amplitude

Comparative study of two methods

Division of wave front	Division of amplitude
The light source is narrow.	Light source is extended.
The wave front emitted by a narrow source is divided in two parts by reflection or refraction.	The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.
The coherent sources obtained are imaginary. **E.g.,** Fresnel's Biprism, Lloyd's mirror, Young's double slit etc.	The coherent sources obtained are real. **E.g.,** Newton's rings, Michelson's Interferometer colours in thin films.

In **Young's double slit** experiment two points of the same wave front are used as two coherent sources where as in **Fresnel's Biprism** two virtual images of same original source are used as two coherent sources.

In **Lloyd's** one original source and its image are used as coherent sources.

Interference of light

- When two light waves of nearly same amplitude, same frequency and traveling in the same direction of medium, superimpose over each other then there occurs variation of intensity of light with distance (maximum and minimum). This phenomenon is defined as interference of light.
- The experiment on interference of light was first performed by Young in 1802.
- The energy or intensity of light gets redistributed non-uniformly as a result of superposition of the light waves.
- Interference is observed in both longitudinal as well as transverse waves.
- In interference phenomenon energy in neither created nor destroyed rather there occurs redistribution of energy in the form of maxima and minima.
- The interference of light takes place in two waves.

  (1) Constructive interference.

  (2) Destructive interference.

Constructive interference

- The resultant amplitude of wave is equal to the sum of amplitudes of individual waves. $A=a_1+a_2$.
- The amplitude of resultant wave is maximum
- The resultant intensity is more than the sum of intensities of individual waves. i.e., $I=I_1+I_2+2\sqrt{I_1I_2}$
- The intensity of resultant wave is maximum i.e., $I_{max}=(\sqrt{I_1}+\sqrt{I_2})^2$.

- The phase difference between two waves is an even multiple of π, i.e., $\phi = 2n\pi$ where n=0, 1, 2..
- The path difference between two waves is an integral multiple of λ; i.e., $x=n\lambda$ where n=0, 1, 2,…
- The time interval between two waves is an even multiple of T/2 i.e., $\tau = 2n\left(\frac{T}{2}\right)$ where n=0, 1, 2…

Destructive interference

- The resultant amplitude of wave is equal to the difference of amplitude of two waves. $A=a_1-a_2$.
- The amplitude of resultant wave is minimum
- The resultant intensity is less than the sum of intensities due individual waves. i.e., $I=I_1+I_2-2\sqrt{I_1I_2}$.
- The intensity of resultant waves is minimum i.e., $I_{min}=(\sqrt{I_1}-\sqrt{I_2})^2$.
- The phase difference between two waves is an odd multiple of π i.e., $\phi = (2n-1)\pi$ where n=1, 2, 3….
- The path difference between two waves is an odd multiple of $\lambda/2$, i.e., $x = (2n-1)\frac{\lambda}{2}$ where n=1, 2,..
- The time interval between two waves is an odd multiple of T/2, i.e., $\tau = (2n-1)\frac{T}{2}$ where n=1, 2, 3...

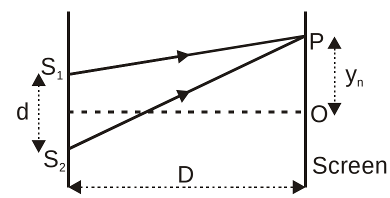
Conditions for sustained interference pattern

- The source of light must be monochromatic.
- Two sources of light must be coherent.
- Frequencies (wavelength, time period) of two waves must be same.
- The amplitudes of two waves must be nearly equal otherwise the contrast between two light sources and the screen must be large.

- The distance between two light sources must be small and the distance between the source and the screen must be large.
- The two coherent sources must be narrow.
- If the two light waves are polarized then their states of polarization must be same.
- The two light waves must travel in the same direction.
- If the source of light is white, then the path difference between the waves emitted by it must be small.
- The vibrations of two waves must be in the same direction.

Shape of the fringes

- The alternate bright and dark strips obtained on the screen as a result of interference are known as interference fringes.
- The interference fringes are **hyperbolic** in shape, because the focus of path difference ($S_2P–S_1P$) for a given value of n is a hyperbola. The foci of this hyperbola are S_1 and S_2.



- Fringes will be straight if D>>d.
- **Fringe of zeroth order** – The fringe obtained at the centre of screen is known as zeroth order fringe. The distance of other fringes are measured from this central fringe.
- The distance of n^{th} bright fringe from central fringe $y_n = \frac{nD\lambda}{d}$ where n=0, 1, 2…
- The distance of n^{th} dark fringe from central fringe $y_n = (2n-1)\frac{D\lambda}{2d}$ where n=0, 1, 2…
- The production of bright or dark fringes depends upon of path difference ($x=S_2P–S_1P$).
- If $x=\frac{yd}{D}=n\lambda$, then bright fringes are produced on the screen.
- If $x=\frac{yd}{D}=(2n-1)\frac{\lambda}{2}$, then dark fringes are produced on the screen.
- The formation of fringes is in accordance with the law of conservation of energy.
- The distance between two consecutive bright or consecutive dark fringes is known as **fringe width** β.

- $\beta = y_{n+1} - y_n = \frac{D\lambda}{d}$.

$A \propto \sqrt{w}$, where w is width of the slit.

Angular fringe width (ω)

- The ratio of fringe width to source screen distance is defined as angular fringe width (ω)

  i.e., $\omega = \frac{\beta}{D} = \frac{\lambda}{d}$.

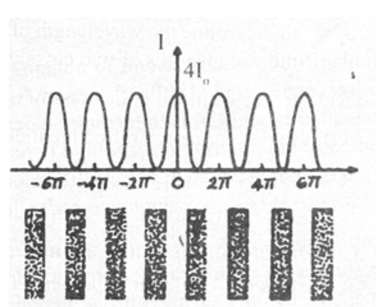
- It has no unit.

Dependence of fringe width

- β depends on the wavelength of light used. i.e., $\beta \alpha \lambda$.
- β depends on the distance between two coherent sources i.e., $\beta \alpha \frac{1}{d}$.
- β depends on the distance of screen from the source i.e., $\beta \alpha D$. If D is very large, then β will also be large and interference pattern will not be observed.

Fringe visibility (V)

- $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$.
- If I_{min}=0, then V=1. In this state the fringe visibility will be maximum (best).
- If I_{max}=0, then V=–1.
- With the help of visibility, knowledge about coherence, fringe contrast and interference pattern is obtained.
- If I_{max}=I_{min}, then V=0. In this condition interference pattern will not be visible.
- **Resultant intensity**
- $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ where ϕ is the phase difference between two waves.
- $I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$



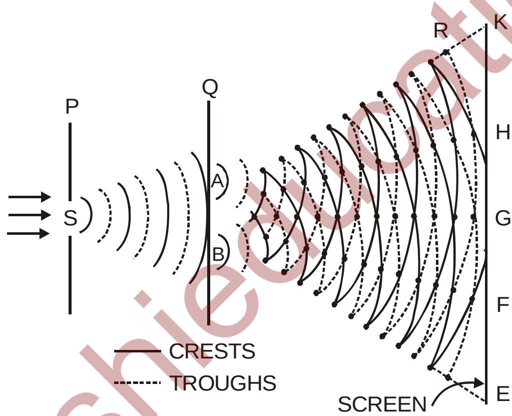
- If $I_1=I_2=I_0$, $I = 2I_0[1+\cos\phi] = 4I_0\cos^2\left(\frac{\phi}{2}\right)$.

Average intensity (I_{av})

- $I_{av} = \frac{I_{max} + I_{min}}{2}$
- $I_{av} = a_1^2 + a_2^2 = I_1 + I_2 = 2I_0$

Young's double slit experiment

- The interference of light was demonstrated for the first time with the help of this experiment.



In this experiment a single slit is exposed to two slits.

- This experiment verifies the wave nature of light.
- The slit S_1 and S_2 behave as coherent sources.
- The bright and dark fringes are obtained alternately.
- The bright fringes are the result of constructive interference whereas dark fringes are the result of destructive interference.
- The central fringe is bright with monochromatic light whereas it is achromatic (white) with white light.
- The formation of fringes is explained on the basis of the Huygens's wave theory of light.

Changes observed in the interference pattern obtained in Young's double slit experiment are

- The fringe width β increases with increase of distance between the source and the screen and vice versa.
- β decreases by increasing distance between two slits S_1 and S_2 and vice versa.
- If the experiment is repeated in water instead of air, then λ decreases and consequently β decrease.
- When S_1 and S_2 both are open, then $I=4a^2= (a_1+a_2)^2$.
- When one of the slits of S_1 and S_2 is close, then $I=a^2=a_1^2=a_2^2$ in this state interference does not take place and uniform illumination is obtained on the screen.
- When one slit is fully open and another one is partially open the contrast between the fringes decreases.
- When the two slits are illuminated by two independent sources then interference fringes are not obtained.
- If a transparent thin film of mica or glass is put in the path of one the slits (waves) then the whole of interference pattern gets shifted towards the side where film is placed. The distance through which the central fringe gets shifted is $\frac{D}{d}(\mu-1)t$.
- When one of the slits is closed and width of another is made of the order of λ, then diffraction fringes are observed.
- When one of the slits is covered with the blue and another one with red transparent papers, the interference pattern is not observed because the wavelengths of two are not same.
- When slit is illuminated with different colours, then fringes are obtained of the same colour but their fringe width is different.
- When the distance between the slits $d<\lambda$ then $\beta>D$ i.e., the fringe pattern will not be visible

$$\left[\beta = \frac{D\lambda}{d} \quad \therefore d<\lambda \quad \therefore \beta > D\right]$$

- When D>>d and white light source is used, then the wavelength absent in front of one of the sources will be $\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}$

Path difference= $(D^2+d^2)–D=\frac{d^2}{2D}$ of the point.

Path difference for dark fringe $\frac{d^2}{2D} = (2n-1)\frac{\lambda}{2}$

Where on putting n=1, 2, 3… the missing wavelengths $\frac{d^2}{D}, \frac{d^2}{3D}$ are obtained.

- If an additional phase difference of π is created in one of the waves then the central fringe becomes dark.