Alternating current

1. An alternating current or e.m.f. is one whose magnitude and direction vary periodically with time.
2. Alternating current abbreviated as ac not A.C or a.c.
3. The simplest types of alternating current and e.m.f. have a sinusoidal variation, given respectively by $i=i_0\sin\omega t$ and $\varepsilon=\varepsilon_0\sin\omega t$ where i_0, ε_0 are called peak values of current and voltage respectively and ω is the angular frequency.
4. The time taken by alternating current to go through one cycle of changes is called its **period (T)** and $T=\frac{2\pi}{\omega}$.
5. The number of cycles per second of an alternating current is called its frequency, $n=\frac{1}{T}=\frac{\omega}{2\pi}$. The phase of an alternating current at any instant represents the fraction of the time period that has elapsed since the current last passed through the zero position of reference. Phase can also be expressed in terms of angle in radians.
6. An alternating current or e.m.f. varies periodically from a maximum in one direction through zero to a maximum in the opposite direction, and so on. The maximum value of the current or e.m.f. in either direction is called the **peak value**.
7. The average or mean value of alternating current or e.m.f. for complete cycle is zero. It has no significance. Hence, the mean value of alternating current ($\bar{i}$) is defined as its average over half a cycle. For positive half cycle $\bar{i}=\frac{1}{\frac{T}{2}}\int_0^{T/2} i\,dt$ where $i=i_0\sin\omega t=\frac{2}{\pi}i_0=0.636i_0$ similarly average value of e.m.f. $\bar{\varepsilon}=\frac{2\varepsilon_0}{\pi}$.
8. The root mean square (r.m.s.) value of an alternating current is the square root of the average of i^2 during a complete cycle where i is the instantaneous value of the alternating current.

(Or)

It is the steady current, which when passed through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time.

(Or)

The r.m.s. velocity of an alternating voltage can be defined as that direct voltage which produces the same rate of heating in a given resistance. The r.m.s. value of alternating voltage is also called as the **effective or the virtual value of the voltage**.

$i_{rms}^2 = \frac{1}{T}\int_0^T i^2\,dt$ Where $i = i_0 \sin \omega t$

$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$

Similarly

$\varepsilon_{rms}^2 = \frac{1}{T}\int_0^T \varepsilon^2\,dt$ where $\varepsilon = \varepsilon_0 \sin \omega t$

$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}}$

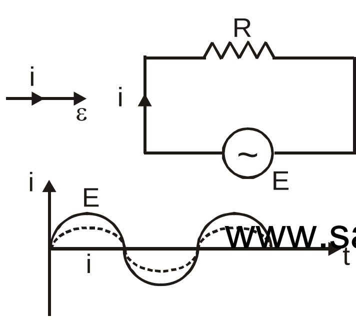
Voltage marked on ac instruments is the r.m.s. voltage, i.e. 220 V ac means $E_{rms} = 220$ V.

9. In any circuit, the ratio of the effective voltage to the effective current is called the **impedance** Z of the circuit. Its unit is ohm.

10. A diagram representing alternating voltage and current as vectors with phase angle between them is called **Phasor diagram**.

11. Purely resistive circuit: A circuit containing an A.C. source and a resistor is known as purely resistive circuit. If $\varepsilon = \varepsilon_0 \sin \omega t$ and the current at a time t is i, then

$\varepsilon_0 \sin \omega t = Ri$



Here both voltage and current are in same phase.

Instantaneous power dissipation

$p = \varepsilon i = \varepsilon_0 i_0 \sin^2 \omega t$

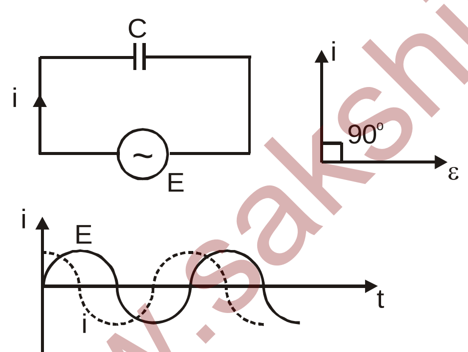
Average power dissipation $\overline{P} = \varepsilon_{rms} i_{rms}$

12. Purely inductive circuit: A circuit containing an A.C source and inductor is known as purely inductive circuit.

If $\varepsilon = \varepsilon_0 \sin \omega t$, the circuit equation is $\varepsilon - L\frac{di}{dt} = 0$; $di = \frac{\varepsilon_0}{L} \sin \omega t dt$ by integration we get

$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ where $i_0 = \frac{\varepsilon_0}{\omega L}$

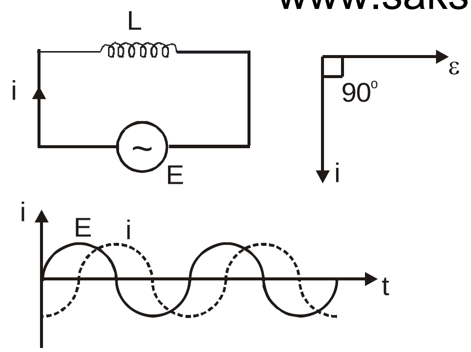
The constant $X_L = \omega L$ plays the role of effective resistance of the circuit. The constant X_L is called the reactance of the inductor. It is zero for direct current (ω=0) and increases as the frequency increases. The current lags the voltage in phase by $\pi/2$ and the quantity ωL is a measure of the effective opposition to the flow of A.C. The average power consumed in a cycle is zero.



13. Purely capacitive circuit: A circuit containing an A.C source and a capacitor is known as purely capacitive circuit. If $\varepsilon = \varepsilon_0 \sin \omega t$, the circuit equation is

Q= $c\varepsilon = c\varepsilon_0 \sin \omega t$ by differentiating;

$i = \frac{dQ}{dt} = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ Where $i_0 = \frac{\varepsilon_0}{\left(\frac{1}{\omega C}\right)}$



The current leads the voltage in phase by $\pi/2$.

The quantity $1/\omega C$ is a measure of the effective opposition of alternating current by a capacitor. It is denoted by X_C and is called **capacitive reactance** $X_C = \frac{1}{\omega C}$.

14. The peak current and the peak e.m.f. in the entire above three circuits can be written as $i_0 = \frac{\varepsilon_0}{Z}$ where Z=R for a purely resistive circuit, $Z=1/\omega C$ for a purely capacitive circuit and $Z=\omega L$ for a purely inductive circuit. The general name for Z is **impedance**.

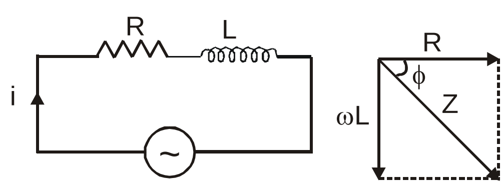
15. If the e.m.f of an A.C circuit is represented by $\varepsilon = \varepsilon_0 \sin \omega t$, the current can be represented as $i = i_0 \sin(\omega t + \phi)$. For purely resistive circuit ϕ=0; for a purely capacitive circuit $\phi = \pi/2$ and for a purely inductive circuit $\phi = \pi/2$. The constant ϕ is called **phase factor**.

16. L–R series circuit

The impedance Z of the circuit is given by $Z=\sqrt{R^2+\omega^2L^2}$.

The current i in the steady state is given by $i = \frac{\varepsilon_0}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t - \phi)$ where $\tan\phi = \left(\frac{\omega L}{R}\right)$

The applied voltage leads the current by $\text{Tan}^{-1}\left(\frac{L\omega}{R}\right)$

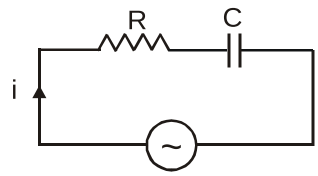


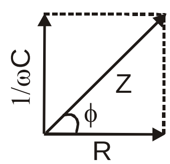
17. R-C series circuit

The impedance Z of the circuit is given by $Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$.

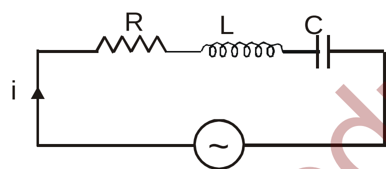
The current i in the steady state is given by $i=\frac{\varepsilon_0}{Z}\sin(\omega t+\phi)$ where

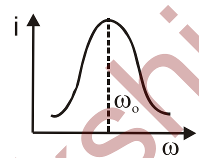
The applied voltage leads the current by $\mathrm{Tan}^{-1}\left(\frac{1}{\omega CR}\right)$

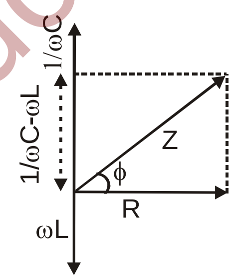




18. LCR series circuit







$\varepsilon_{rms}=\sqrt{\varepsilon_R^2+(\varepsilon_L\sim\varepsilon_C)^2}$

$Z=\sqrt{R^2+(X_L\sim X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$

$\tan\phi=\frac{L\omega-1/\omega C}{R}$; $\omega_0=\frac{1}{\sqrt{LC}}$

(i) When $L\omega>\frac{1}{\omega C}$, $\tan\phi$ is positive i.e., ϕ is positive in such case e.m.f leads the current.

(ii) When $L\omega<\frac{1}{\omega C}$, $\tan\phi$ is negative i.e., ϕ is negative in such case e.m.f lags behind the current.

(iii) When $L\omega = \frac{1}{\omega C}$, $\tan\phi$ is zero i.e., ϕ is zero in such case current and e.m.f are in phase with each other.

When $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$ the impedance becomes minimum and hence current will be maximum. The circuit is then said to be resonance and the corresponding frequency is known as **resonant frequency**. The resonant frequency= $\frac{1}{2\pi}.\frac{1}{\sqrt{LC}}$. The peak current in this case is $\frac{\varepsilon_0}{R}$

19. Quality factor of resonance: the selectivity or sharpness of resonant circuit is measured by Q-factor called **quality factor**.

The Q factor or quality factor of a resonant LCR – circuit is defined as ratio of the voltage drop across inductor (or capacitor) to the applied voltage.

$Q = \frac{\text{volt age acros L(or C)}}{\text{applied voltage}}$ $\quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

The Q-factor of LCR series circuit will be large (or more sharpness) if R is low or L is large or C is low.

20. Power in A.C. circuit: The average power P delivered by A.C source in a complete cycle is given by P= $\varepsilon_{rms}.i_{rms}\cos\phi$ where $\cos\phi$ is called the power factor of LCR circuit. P also represents the average power delivered in a long time.

21. <u>Advantages of AC over DC</u>

1. The generation of AC is more economic than DC.
2. AC voltages can be easily stepped up or stepped down using transformers.
3. AC can be transmitted to longer distances with less loss of energy.
4. AC can be easily converted into DC by using rectifiers.

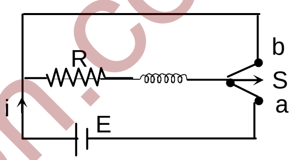
22. <u>Disadvantages</u>

1. AC is more fatal and dangerous than DC.
2. AC always flows on the outer layer of the conductor (skin effect) and hence AC requires stranded wires.

3. AC cannot be used in electrolysis like electroplating etc.

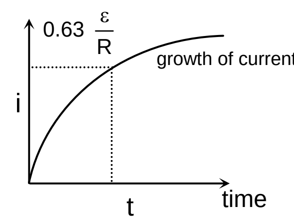
DC Circuits

Growth of Current in LR Circuit



1. When switch "S" is closed at t=0, $\varepsilon - L\frac{di}{dt} = Ri$.

2. At time t, current $i = \frac{\varepsilon}{R}\left(1 - e^{-t\frac{R}{L}}\right)$.



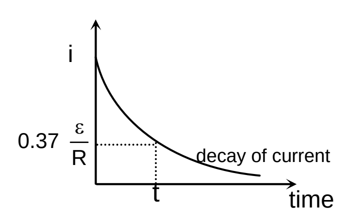
3. The constant L/R has dimensions of time and is called the **inductive time constant** (τ) of the LR circuit.

4. $t = \tau;\ i = 0.63 i_0$, in one time constant, the current reaches 63% of the maximum value. The time constant tells us how fast the current will grow.

5. $i = i_0$, when $t = \infty$, where $i = \frac{\varepsilon}{R}$.

   Theoretically current grows to maximum value after infinite time. But practically it grows to maximum after 5τ.

Decay of current



6. When switch "S" is open at t=0; $-L\frac{di}{dt} = Ri$

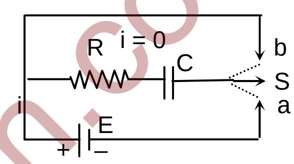
   at t=0, $i = i_0$

   at time t, $i = i_0 e^{-\frac{t}{\tau}}$

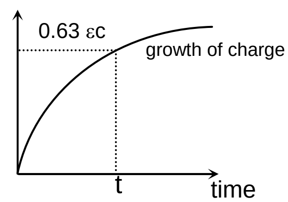
The current reduces to 37% of the initial value in one time constant i.e., 63% of the decay is complete.

7. Energy stored in inductor $E = \frac{1}{2}Li^2$.

Charging of a capacitor



8. When a capacitor is connected to a battery, positive charge appears on one plate and negative charge on the other. The potential difference between the plates ultimately becomes equal to e.m.f of the battery. The whole process takes some time and during this time there is an electric current through connecting wires and the battery.



9. Using Kirchhoff's loop law $\frac{q}{C} + Ri - \varepsilon = 0$.

10. At any time t, $q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{CR}}\right)$

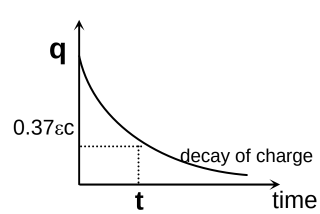
$V = E\left(1 - e^{-\frac{t}{CR}}\right)$; $i = i_0 e^{-\frac{t}{CR}}$.

11. The constant RC has dimensions of time and is called **capacitive time constant** (τ).

12. In one time constant (τ=RC), the charge accumulated on the capacitor is $q = 0.63\,\varepsilon C$.

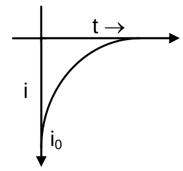
Discharging of a capacitor

13. When the plates of a charged capacitor are connected through a conducting wire, the capacitor gets discharged, again there is a flow of charge through the wires and hence there is a current



14. $\frac{q}{C} - Ri = 0$

15. $q = Qe^{-\frac{t}{RC}}$, where $Q = \varepsilon C$



$V = E\,e^{-\frac{t}{CR}}$; ; $i = -i_0\, e^{-\frac{t}{CR}}$.

16. At t=RC, q=0.37Q, i.e., 63% of the discharging is complete in one time constant.