Vector Addition

1. **Scalar:** A physical quantity having only magnitude but no direction is called a scalar.

   eg: Time, mass, distance, speed, electric charge, etc.

2. **Vector:** A physical quantity having both magnitude and direction and which obeys the laws of vector addition is called a vector quantity.

   eg: Displacement, velocity, acceleration, intensity of electric field, etc.

3. Surface area can be treated both as a scalar and a vector. A is magnitude of surface area which is a scalar. If $\hat{n}$ is a unit vector normal to the surface, we can write $A\hat{n}$ as a vector.

4. Electric current and velocity of light are not vectors even though they have direction since they do not obey the laws of addition.

5. A vector quantity which has direction by its nature is called a polar vector. Ex: velocity.

6. A vector quantity which has direction by a convention is called a pseudo (or) axial (or) non-polar vector. The direction of pseudo vector can be known from right hand thumb rule. Ex: Angular velocity.

7. **Equal vectors:** Vectors having same magnitude and which have same direction are called equal vectors. Their corresponding components are equal.

8. **Negative vectors:** A vector which has the same magnitude as that of another and which is opposite in direction is called a negative vector.

9. **Null Vector (Zero Vectors):** A vector whose magnitude is zero and which has no specific direction is called a null vector.

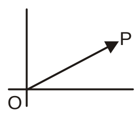
   e.g. 1) The cross product of two parallel vectors is a null vector.

   2) The difference of two equal vectors is a null vector.

10. **Unit vector:** It is a vector whose magnitude is unity. A unit vector parallel to a given vector.

    If $\vec{A}$ is a vector, the unit vector in the direction of $\vec{A}$ is written as $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$. $\hat{i}, \hat{j}$ and $\hat{k}$ are units vectors along x, y and z axis.

11. **Position vector:** The position of a particle is described by a position vector which is drawn from the origin of a reference frame.The position vector of a particle P in space is given by $\overline{OP} = \bar{r} = xi + yj + zk$.



Its magnitude is given by $r = \sqrt{x^2 + y^2 + z^2}$

Unit vector of $\bar{r}$ is given by $r = \frac{\bar{r}}{|\bar{r}|} = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$

Addition of Vectors

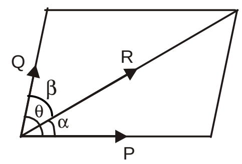
12. Resultant can be found by using

a) Triangle law of vectors b) Parallelogram law of vectors c) Polygon law of vectors

13. **Triangle law:** If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side taken in the reverse order represents their sum or resultant both in magnitude and direction.

14. **Parallelogram law**

If two vectors $\vec{P}$ and $\vec{Q}$ are represented by the two sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\text{Tan}\,\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$ and $\tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}$

15. If the resultant $\bar{R}$ of $\bar{P}$ and $\bar{Q}$ makes an angle α with $\bar{P}$ and β with $\bar{Q}$ and if $\bar{P} > \bar{Q}$ then $\alpha < \beta$.

16. For two vectors $\bar{P}$ and $\bar{Q}$, $R_{max} = P + Q$ and $R_{min} = P - Q$

17. If two vectors $\bar{P}$ and $\bar{Q}$ have equal magnitudes x, then $R = 2x\cos\frac{\theta}{2}$

18. Vectors addition obeys

a) Commutative law: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

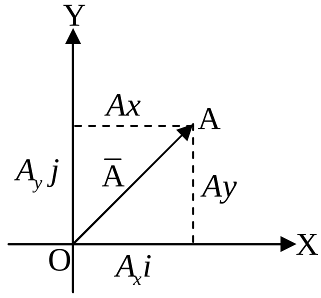
b) Associative law: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

c) Distributive law: $m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$ where m is a scalar.

19. **Polygon law:** If a number of vectors are represented by the sides of a polygon taken in the same order, the resultant is represented by the closing side of the polygon taken in the reverse order.

20. Resolution of a vector

Consider a vector $\bar{A}$ represented along $\overrightarrow{OA}$ in two co-ordinate system, which makes an angle θ with X-axis.



$$A = A_x i + A_y j \quad \text{and} \quad |\bar{A}| = \sqrt{A_x^2 + A_y^2}$$

$$Cos\theta = \frac{Ax}{|\bar{A}|} \Rightarrow A_x = |\bar{A}| Cos\theta$$

$$Sin\theta = \frac{A_y}{|\bar{A}|} \Rightarrow A_y = |\bar{A}| Sin\theta$$

If the vector makes an angle α with X-axis, β with Y axis and γ with Z-axis Then

$$A = A_x i + A_y j + A_z k \quad \text{and} \quad |\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$Cos\alpha = \frac{A_x}{|\bar{A}|} = \ell \,;\, Cos\beta = \frac{A_y}{|\bar{A}|} = m \quad \text{and} \quad Cos\gamma = \frac{A_z}{|\bar{A}|} = m$$

$$Cos\alpha^2 + Cos^2\beta + Cos^2\gamma = 1 \quad \text{(or)} \quad \ell^2 + m^2 + n^2 = 1$$

And $Sin^2\alpha + Sin^2\beta + Sin^2\gamma = 2$ (Law of cosines)

21. If ℓ_1, m_1, n_1 and $\ell_2\ m_2\ n_2$ the direction cosines of two vectors and θ is the angle between them then $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$.

22. Component of a vector is a vector.

23. If vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are parallel, then $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$ and $\vec{A} = K\vec{B}$ where K is a scalar.

Equilibrium

24. Equilibrium is the state of a body in which there is no acceleration i.e., net force acting on a body is zero.

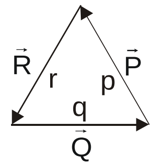
25. The forces whose lines of action pass through a common point are called concurrent forces.

26. Resultant force is the single force which produces the same effect as a given system of forces acting simultaneously.

27. A force which when acting along with a given system of forces produces equilibrium is called the equilibrant.

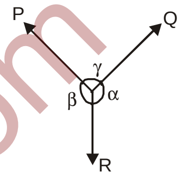
28. Resultant and equilibrant have equal magnitude and opposite direction. They act along the same line and they are themselves in equilibrium.

29. Triangle law of forces: If a body is in equilibrium under the action of three coplanar forces, then these forces can be represented in magnitude as well as direction by the three sides of a triangle taken in order. $\frac{p}{|\vec{P}|} = \frac{q}{|\vec{Q}|} = \frac{r}{|\vec{R}|}$



Where p, q, r are sides of a triangle. $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors.

30. Lami's theorem: When three coplanar forces $\vec{P}, \vec{Q}$ and $\vec{R}$ keep a body in equilibrium, then $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$.



31. If $\vec{A}_1 + \vec{A}_2 + \vec{A}_3 + ... + \vec{A}_n = 0$ and $A_1 = A_2 = A_3 = \ldots A_n$, then the adjacent vectors are inclined to each other at an angle $\frac{2\pi}{N}$ or $\frac{360°}{N}$.

32. N forces each of magnitude F are acting on a point and angle between any two adjacent forces is □, then resultant force $F_{resultant} = \frac{F \sin\left(\frac{N\theta}{2}\right)}{\sin(\theta/2)}$.

33. Body Pulled Horizontally

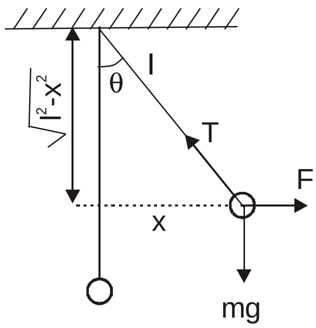
The horizontal force required to pull a suspended body through an angle θ with the vertical is given by

$T\sin\theta = F$ and $T\cos\theta = mg$

$T^2 = F^2 + (mg)^2$

$Tan\theta = \frac{F}{mg}$

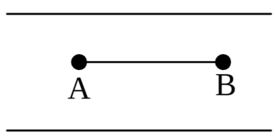
$\Rightarrow F = mg\,Tan\theta$



Motion of the boat in a river

34. Let V_B be the velocity of the boat and V_R the velocity of the river.

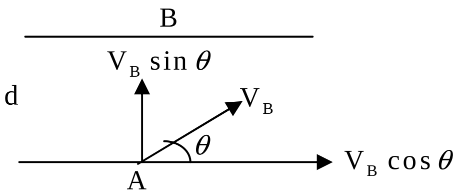
1. The time taken by the boat to go from A to B and B to A in still water $T = \frac{2d}{V_B}$



2. If the river is flowing

$t_{down} = \dfrac{d}{V_B + V_R}$ and $t_{up} = \dfrac{d}{V_B - V_R}$

$T = t_d + t_u = \dfrac{2d\,V_B}{V_B^2 - V_R^2}$

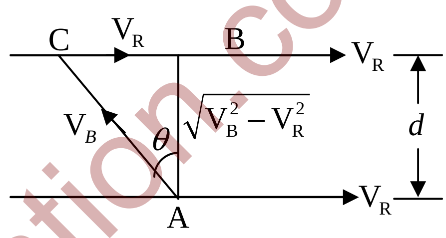


3. $d = (V_B\ Sin\theta)t$

Time taken by the boat to cross the river is $t = \dfrac{d}{V_B\ Sin\theta}$

If $\theta = 90^o$, then t is minimum .i.e. the boat can cross the river in a shortest time if it moves along AB.

35. Shortest Path



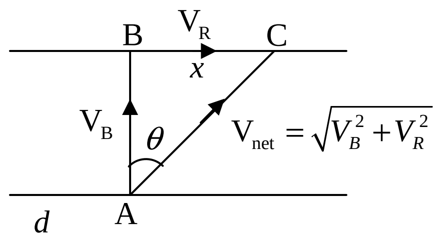
$Sin\theta = \dfrac{V_R}{V_B}$ Or $\theta = Sin^{-1}\left(\dfrac{V_R}{V_B}\right)$

And $90 + Sin^{-1}\left(\dfrac{V_R}{V_B}\right)$ with stream

Resultant velocity = $\sqrt{V_B^2 - V_R^2}$

Time taken to cross the river $t = \dfrac{d}{\sqrt{V_B^2 - V_R^2}}$

36. Shortest time



Resultant velocity = $\sqrt{V_B^2 + V_R^2}$

Time taken to cross the river is $t = \dfrac{d}{V_B}$

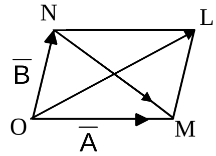
Also $Tan\theta = \dfrac{V_R}{V_B} = \dfrac{x}{d} \Rightarrow x = \dfrac{V_R}{V_B}d$

37. Subtraction of two vectors

a) If $\vec{P}$ and $\vec{Q}$ are two vectors, then $\vec{P} - \vec{Q}$ is defined as $\vec{P} + (-\vec{Q})$ where $-\vec{Q}$ is the negative vector of $\vec{Q}$.

If $\vec{R} = \vec{P} - \vec{Q}$, then $R = \sqrt{P^2 + Q^2 - 2PQCos\theta}$

In the parallelogram OMLN, the diagonal OL represents $\vec{A} + \vec{B}$ and the diagonal NM represents $\vec{A} - \vec{B}$



b) subtraction of vectors does not obey commutative law $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

c) subtraction of vectors does not obey Associative law $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

d) Subtraction of vectors obeys distributive law $m(\vec{A} - \vec{B}) = m\vec{A} - m\vec{B}$.

38. For two equal vectors $R = 2x\, Sin\left(\frac{\theta}{2}\right)$