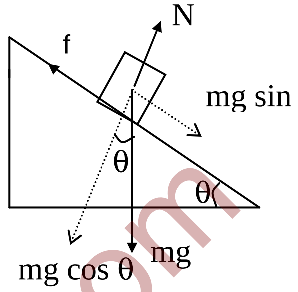
Friction

Inclined Plane

1. For a block of mass m on an inclined plane of inclination θ with the horizontal,



a) If Applied force < frictional force

Or $mg \sin\theta < f$ Or $mg\sin\theta < \mu_s mg\cos\theta$

Or $Tan\theta < \mu_s$ Or $Tan\theta < Tan\alpha$ $\Rightarrow \theta < \alpha$ Then the block doesn't slide. Since the static friction is rest adjusting,

Frictional force = $mg\sin\theta$

b) If Applied force = frictional force

$mg\sin\theta = f$ (or), if $\theta = \alpha$, then the body tends to move (or) ready to move. The angle of inclination in this condition is called the angle of repose. Angle of repose in independent of the weight of the body

Frictional force = $mg\sin\theta = \mu_s mg\cos\theta$

$\mu_s = Tan\alpha$

c) If Applied force > frictional force

Or $mg\sin\theta > \mu_s N$ Or $mg\sin\theta > \mu_s mg\cos\theta$ or $Tan\theta > \mu_s$ Or $\theta > \alpha$

Now, the frictional force = $f_k = \mu_k N$

$\therefore f_k = \mu_k mg\cos\theta$

The resultant force on the block is given by

$ma = mg(\sin\theta - \mu_k\cos\theta)$ Or $a = g(\sin\theta - \mu_k\cos\theta)$

This is the acceleration of the body sliding down the inclined plane.

2. Sliding down the inclined plane

a) If $\theta \geq \alpha$, the block slides down with an acceleration given by $a = g[\sin\theta - \mu_k\cos\theta]$.

b) If $\theta \geq \alpha$, and the block slides down from the top of the inclined plane. Velocity at the bottom of the plane is

$V = \sqrt{2gl(\sin\theta - \mu_k \cos\theta)}$.

c) In the above case time of descent is $t = \sqrt{\frac{2L}{g(\sin\theta - \mu_k \cos\theta)}}$.

3. Moving up the inclined plane

a) If a block is projected up a rough inclined plane, the acceleration of the block is

$a = -g[\sin\theta + \mu_k \cos\theta]$.

b) Force opposing the motion of the block is

$F = mg\sin\theta + \mu_k mg\cos\theta$.

c) The distance traveled by the block up the plane before the velocity becomes zero is

$S = \frac{u^2}{2g(\sin\theta + \mu_k \cos\theta)}$.

d) The time of ascent is $t = \frac{u}{g(\sin\theta + \mu_k \cos\theta)}$. In the above case the block will come down sliding only if $\theta \geq \alpha$.

e) In the above case if time of decent is n times the time of ascent, then

$\mu = \tan\theta \left[\frac{n^2 - 1}{n^2 + 1}\right]$.

f) Force needed to be applied parallel to the plane to move the block up with constant velocity is

$F = mg(\sin\theta + \mu_k \cos\theta)$.

g) Force needed to be applied parallel to the plane to move the block up with an acceleration a is

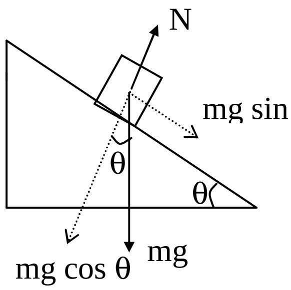
$F = mg(\sin\theta + \mu_k \cos\theta) + ma$.

h) If block has a tendency to slide, the force to be applied on the block parallel and up the plane to prevent the block from sliding is $F = mg(\sin\theta - \mu_s \cos\theta)$.

4. Smooth inclined plane

a) Normal reaction (N) = $mg\cos\theta$

a) Acceleration of sliding block $a = g\sin\theta$



b) If l is the length of the inclined plane and h is the height. The time taken to slide down starting from rest from the top is

$$t = \sqrt{\frac{21}{g\sin\theta}} = \sqrt{\frac{2h}{g\sin^2\theta}}.$$

d) Velocity of the block at the bottom of the inclined plane is

$V = \sqrt{2gl\sin\theta} = \sqrt{2gh}$ same as the speed attained if block falls freely from the top of the inclined plane.

e) Distance traveled up the plane before its velocity becomes zero is

$$S = \frac{u^2}{2g\sin\theta}.$$

5. If the time taken by a block to slide downs a rough inclined plane of angle θ is an n time that on identical smooth inclined plane. Then μ of the rough plane is $\mu = \left(1 - \frac{1}{n^2}\right) Tan\theta$.

6. Sand is piled up on a horizontal ground in the form of a regular cone of a fixed base of radius R. The maximum volume of the cone without the sand collapsing is $V_{\max} = \frac{1}{3}\frac{\pi h^3}{\mu^3}$.

7. An inclined plane of inclination θ is upper half smooth and lower half rough. A body starts sliding from the top from rest and comes to rest at the bottom. If the coefficient of friction of the lower half is μ, then $\mu = 2\tan\theta$.