Vertical Circular Motion

1. A body of mass m is rotated in a vertical circle of radius r.

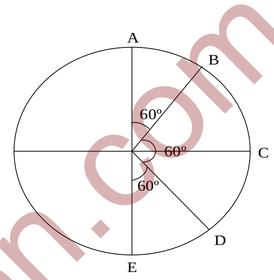
$A \rightarrow v_1 = \sqrt{gr}; T_1 = 0$

$B \rightarrow v_2 = \sqrt{2gr}; T_2 = 1.5mg$

$C \rightarrow v_3 = \sqrt{3gr}; T_3 = 3mg$

$D \rightarrow v_4 = \sqrt{4gr}; T_4 = 4.5mg$

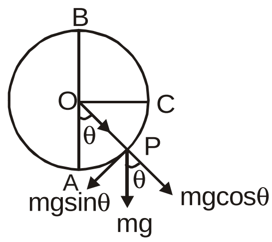
$E \rightarrow v_5 = \sqrt{5gr}; T_5 = 6mg$



2. If a body of mass m is rotated in a vertical circle of radius r the velocity and tension at any point of the vertical circle is given by T

a) Speed of the particle $u^2 - 2gr(1 - Co$ v = $\sqrt{u^2 - 2gr(1 - Cos\,\theta)}$

Where u is the velocity at the lowest point



b) Centripetal force $mv^2 / r = T - mg \cos\theta$

c) Tangential acceleration = $g \sin\theta$

d) Tangential force = $mg \sin\theta$

e) Tension in the string $T = mv^2/r + mg\cos\theta = \frac{m}{r}[v^2 + g(r-h)]$

f) Velocity, speed, K.E, linear momentum, angular momentum, angular velocity, all are variables. Only total energy remains constant

g) If $u < \sqrt{2gr}$ the body oscillates about A.

h) If $0 < u < \sqrt{2gr}$; $0 < \theta < 90°$.

i) If $\sqrt{2gr} < u < \sqrt{5gr}$ the body leaves the path without completing the circle.

j) A body is projected with a velocity 'u' at the lowest point

i) Height at which velocity u = 0 is $h = u^2/2g$

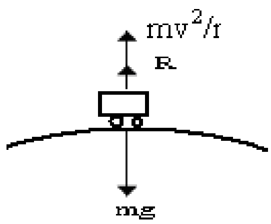
ii) Height at which Tension T = 0 is $h = \frac{u^2 + rg}{3g}$

k) Angle with vertical at which velocity v = 0 is $\cos\theta = 1 - \frac{u^2}{2gr}$

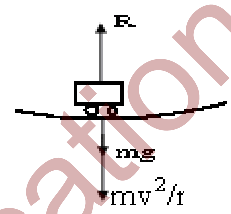
l) Angle with vertical at which the tension T = 0 is $\cos\theta = 2/3 - u^2/3gr$

3. **Reaction on the road**

Convexity up:



Concavity up:



$$R = mg - \frac{mv^2}{r}$$

$$R = mg + \frac{mv^2}{r}$$

4. A particle is freely sliding down from the top of a smooth convex hemisphere of radius r.

a. The particle is ready to leave the surface at a vertical distance h = r/3 from the highest point.

b. If the position vector of the particle with respect to the centre of curvature makes an angle θ with vertical then Cosθ = 2/3 and $\alpha = \cos^{-1}\frac{2}{3} = 48°11$

c. Velocity = $\sqrt{\frac{2gR}{3}}$

d. If the body starts from initial velocity, then $\cos\theta = \left[\frac{u^2}{3rg} + \frac{2}{3}\right]$

5. A body of mass m is sliding along an inclined plane from a vertical height h as shown in the figure. For the body to describe a vertical circle of radius R, the minimum height in terms of R is given by $h = \frac{5R}{2}$.