## Springs

1. A spring of force constant $K$ is cut into two parts whose lengths in the ratio 2: 3 . The part having larger force constant is
1) Shorter spring
2) Longer spring
3) Both have equal force constants
4) Nothing can be said
2. The time period of stiffer spring as compared to that of a soft spring is
1) Less
2) More
3) Infinity
4) Depends upon the spring material
3. When a block is suspended by means of spring the restoring force per unit displacement of the block is called
1) Force Constant
2) Pressure
3) Stress
4) Strength of spring
4. One of the two clocks on earth is controlled by a pendulum and other by a spring. If both the clocks be taken to the moon, then which clock will show same time as that on earth?
1) Spring clock
2) Pendulum clock
3) Both ' 1 ' and ' 2 '
4) Neither ' 1 ' and '2'
5. (A): A loaded spring oscillating with frequency ' $f$ ' continues to maintain the same frequency in an earth satellite.
(R): Frequency of loaded spring is independent of acceleration due to gravity.
1) $A$ and $R$ are true but $R$ is not correct explanation for $A$.
2) $A$ and $R$ are true and $R$ is correct explanation for $A$.
3) $A$ is true but $R$ is false.
4) $A$ is false but $R$ is true.
6. A horizontal platform undergoes SHM in the vertical direction. A coin is placed on the platform. The angular velocity of oscillations is increased, the coin may lose contact with the platform when
(a) The platform is at the mean position of oscillation
(b)The platform is the highest position of oscillation
(c) The angular velocity is $\sqrt{2 g / A}$
(d) The angular velocity is $\sqrt{g / A}$
(1) b, d
(b) a, d
(c) $\mathrm{a}, \mathrm{c}$
(d) $\mathrm{b}, \mathrm{c}$
7. A): A loaded spring oscillating with frequency ' $f$ ' continues to maintain the same frequency in an earth satellite.
$R$ ): Frequency of loaded spring is independent of acceleration due to gravity.
1) A and $R$ true and $R$ is correct explanation for A.
2) $A$ and $R$ are true and $R$ is not correct explanation for $A$.
3) $A$ is true $R$ is false.
4) A is false $R$ is true.
8. A): When a spring is cut into two equal parts, spring constant of each piece is doubled.
R): Spring constant is inversely proportional to length of spring.
1) $A$ and $R$ true and $R$ is correct explanation for $A$.
2) $A$ and $R$ are true and $R$ is not correct explanation for $A$.
3) $A$ is true $R$ is false.
4) A is false $R$ is true.
9. The total force constant of the springs shown in the figure will be

1) $\left(\frac{k_{1}}{2}+k_{2}\right)$
2) $\left(\frac{1}{2 k_{1}}+\frac{1}{k_{2}}\right)^{-1}$
3) $\left(\frac{1}{2 k_{1}}+\frac{1}{k_{2}}\right)$
4) $\left(2 k_{1}+\frac{1}{k_{2}}\right)$
10. The free end of a spring hanging from the rigid support, a block of mass ' $m$ ' is hung and slowly allowed to come to its equilibrium position. Then the stretching in the spring is‘d’. If the same block is attached to same spring and allowed to fall suddenly the amount of stretching is [force constant K]
1) $\mathrm{mg} / \mathrm{k}$
2) $2 d$
3) $\frac{m g}{3 k}$
4) $4 d$
11. An ideal spring with a spring constant ' $K$ ' is hung from the ceiling and a block of mass ' $m$ ' is attached to its lower end. The mass is released with spring initially un-stretched. Then the maximum extension in the spring is
1) $4 \mathrm{mg} / \mathrm{K}$
2) $2 \mathrm{mg} / \mathrm{K}$
3) $\mathrm{mg} / \mathrm{K}$
4) $\mathrm{mg} / 2 \mathrm{~K}$
12. A mass ' $M$ ' is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period ' $T$, if the mass is increased by ' $m$ ', time period becomes $\frac{5 T}{3}$, then the ratio of ( $\mathrm{m} / \mathrm{M}$ ) is
1) $\frac{3}{2}$
2) $\frac{25}{9}$
3) $\frac{16}{9}$
4) $\frac{5}{3}$
13. Two springs of force constants $1000 \mathrm{~N} / \mathrm{m}$ and $2000 \mathrm{~N} / \mathrm{m}$ are stretched by same force. The ratio of their respective potential energies is
1) $2: 1$
2) $1: 2$
3) $4: 1$
4) $1: 4$
14. The elongation of a spring of length ' $L$ ' and of negligible mass due to a force is ' $x$ '. The spring is cut into two pieces of length in ratio $1: n$. The ratio of the respective spring constants is
1) $n: 1$
2) $1: n$
3) $n^{2}: 1$
4) $1: n^{2}$
15. Two springs with stiffness constants $K_{1}$ and $K_{2}\left(K_{2}>K_{1}\right)$ are stretched by the same force. More work is done on the spring with spring constant
1) $K_{1}$
2) $\mathrm{K}_{2}$
3) Same for both
4) None
16. The natural angular frequency of a particle of mass ' $m$ ' attached to an ideal spring of force constant ' $K$ ' is
1) $\sqrt{\frac{K}{m}}$
2) $\sqrt{\frac{\mathrm{m}}{\mathrm{K}}}$
3) $\left(\frac{K}{m}\right)^{2}$
4) $\left(\frac{\mathrm{m}}{\mathrm{K}}\right)^{2}$
17. Springs of spring constants $K, 2 K, 4 K, 8 k \ldots 2048 \mathrm{~K}$ is connected in series. A mass ' $m$ ' is attached to one end the system is allowed to oscillation. The time period is approximately
1) $2 \pi \sqrt{\frac{m}{2 K}}$
2) $2 \pi \sqrt{\frac{2 m}{K}}$
3) $2 \pi \sqrt{\frac{\mathrm{~m}}{4 \mathrm{~K}}}$
4) $2 \pi \sqrt{\frac{4 \mathrm{~m}}{\mathrm{~K}}}$
18. Two masses $m$ and $M$ are suspended together by a mass less spring of force constant ' $K$ '. When the masses are in equilibrium, $M$ is removed without disturbing the system. The amplitude of oscillation is
1) $\frac{\mathrm{Mg}}{\mathrm{K}}$
2) $\frac{\mathrm{mg}}{\mathrm{K}}$
3) $\frac{(M+m) g}{K}$
4) $\frac{(M-m) g}{K}$
19. If a spring of force constant ' $K$ ' is cut into three equal parts, then the force constant of each part will be
1) $\mathrm{K} / 3$
2) K
3) 3 K
4) 6 K
20. Three masses $0.1 \mathrm{~kg}, 0.3 \mathrm{~kg}$ and 0.4 kg are suspended at the end of the spring. When the 0.4 kg is removed, the system oscillates with a period of ' 2 ' sec. When $0.3 \mathbf{k g}$ mass is also removed, the system will oscillate with a period
1) 1 sec
2) 2 sec
3) 3 sec
4) 4 sec
21. A spring of spring constant $5^{\prime} 10^{3} \mathrm{~N} / \mathrm{m}$ is stretched initially by 5 cm from the un-stretched position. Then the work required to stretch it further by $\mathbf{5 c m}$ is
1) 12.50 Nm
2) 18.75 Nm
3) 25.00 Nm
4) 6.25 Nm
22. On a smooth inclined plane, a body of mass $M$ is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant $K$, the period of oscillation of the body (assuming the springs as mass less) is
1) $2 \pi\left(\frac{M}{2 K}\right)^{1 / 2}$
2) $2 \pi\left(\frac{2 M}{K}\right)^{1 / 2}$
3) $2 \pi \frac{M g \sin \theta}{2 K}$
4) $2 \pi\left(\frac{2 M g}{K}\right)^{1 / 2}$
23. Two identical balls $A$ and $B$ each of mass 0.1 kg are attached to two identical mass less springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centers of the balls can move in a circle of radius $\mathbf{0 . 0 6} \mathbf{~ m}$. Each spring has a natural length of $0.06 \pi \mathrm{~m}$ and force constant $0.1 \mathrm{~N} / \mathrm{m}$. Initially both the balls are displaced by an angle $\theta=\pi / 6$ radian with respect to the diameter $P Q$ of the circle and released from rest. The frequency of oscillation of the ball $B$ is
1) $\pi \mathrm{Hz}$
2) $\frac{1}{\pi} \mathrm{~Hz}$
3) $2 \pi \mathrm{~Hz}$
4) $\frac{1}{2 \pi} \mathrm{~Hz}$

24. Three masses $700 \mathrm{~g}, 500 \mathrm{~g}$, and 400 g are suspended at the end of a spring a shown and are in equilibrium. When the 700 g mass is removed, the system oscillates with a period of 3 seconds, when the 500 gm mass is also removed; it will oscillate with a period of
1) 1 s
2) 2 s
3) 3 s
4) $\sqrt{\frac{12}{5}} \mathrm{~s}$
25. A particle of mass $m$ is attached to three identical springs $A, B$ and $C$ each of force constant $\boldsymbol{k}$ a shown in figure. If the particle of mass $\boldsymbol{m}$ is pushed slightly against the spring $A$ and released then the time period of oscillations is
1) $2 \pi \sqrt{\frac{2 m}{k}}$
2) $2 \pi \sqrt{\frac{m}{2 k}}$
3) $2 \pi \sqrt{\frac{m}{k}}$
4) $2 \pi \sqrt{\frac{m}{3 k}}$

> Key

1) 1
2) 1
3) 1
4) 1
5) 2
6) 1
7) 1
8) 1
9) 3
10) 2
11) 2
12) 3
13) 1
14) 1
15) 1
16) 1
17) 2
18) 2
19) 3
20) 1
21) 2
22) 1
23) 2
24) 2
25) 2

## Hints

9. $k_{1} / k_{2} \Rightarrow 2 k_{1}$
$2 \mathrm{k}_{1}$ is series to $\mathrm{k}_{2}$
$\frac{1}{k}=\frac{1}{k_{1}}+\frac{1}{k_{2}}$
$k=\left(\frac{1}{2 k_{1}}+\frac{1}{k_{2}}\right)$
10. $d=\frac{m g}{k} \rightarrow 1$

When the load is suddenly released then loss gravitational $\mathrm{PE}=$ gain in elastic PE
$m g x=\frac{1}{2} k x^{2}$
$x=\frac{2 m g}{k} \rightarrow 2$
From (1) and (2) $x=2 d$
11. Loss of gravitational $\mathrm{PE}=$ gain is elastic PE
$\operatorname{mgx}=\frac{1}{2} k x^{2}$
$x=\frac{2 m g}{k}$
12. $T \propto \sqrt{m}$

$$
\frac{T_{1}}{T_{2}}=\sqrt{\frac{m_{1}}{m_{2}}}
$$

$$
\frac{T}{(5 T / 3)}=\sqrt{\frac{m}{m+m}}
$$

$$
\frac{3}{5}=\sqrt{\frac{m}{m+m}}
$$

$\frac{9}{25}=\frac{m}{m+m}$
$\frac{m}{m}=\frac{16}{9}$
13. P.E. $U=\frac{1}{2} \frac{F^{2}}{k}$
$U \propto \frac{1}{K}$
$\frac{U_{1}}{U_{2}}=\frac{k_{2}}{19}=\frac{2000}{1000}=\frac{2}{1}$
14. P.E. $U=\frac{1}{2} \frac{F^{2}}{k}$
$\mathrm{F}=\cos ^{2}$
$U \propto \frac{1}{K}$
$\frac{U_{1}}{U_{2}}=\frac{k_{2}}{19}=\frac{2000}{1000}=\frac{2}{1}$
15. $W=\frac{1}{2} \frac{F^{2}}{k}=$ constant
$W \propto \frac{1}{k}$
$\mathrm{K}_{1}<\mathrm{K}_{2}$
$W_{1}>W_{2}$
16. $F=-k x$
$m a=-k x$
$a=-\frac{k}{m} \cdot x$
$a=-\omega^{2} \cdot x$
$\omega^{2}=\frac{k}{m}$

$$
W=\sqrt{\frac{k}{m}}
$$

17. $\frac{1}{k_{g}}=\frac{1}{k}+\frac{1}{2 k}+\frac{1}{4 k}+\frac{1}{8 k}+\ldots$

$$
\begin{aligned}
& \frac{1}{k_{g}}=\frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots .\right] \\
& \frac{1}{k_{g}}=\frac{1}{k}\left[\frac{1}{1-\frac{1}{2}}\right]=\frac{2}{k} \\
& \mathrm{~K}_{\mathrm{S}}=\mathrm{k} / 2 \\
& T=2 \pi \sqrt{\frac{m}{k_{s}}} \\
& T=2 \pi \sqrt{\frac{2 m}{k}}
\end{aligned}
$$

18. Amplitude oscillation $=\frac{m g}{k}$
19. $K \propto \frac{1}{l}$
20. $\sin c e T=2 \pi \sqrt{\frac{m}{k}}$

$$
\begin{aligned}
& 2=2 \pi \sqrt{\frac{0.1+0.3}{k}} \\
& 2=2 \pi \sqrt{\frac{0.4}{k} \rightarrow 1}
\end{aligned}
$$

When 0.3 is removed then
$T=2 \pi \sqrt{\frac{0.1}{k}} \rightarrow 2$
$\frac{(1)}{(2)} \frac{2}{T}=\sqrt{\frac{0.4}{0.1}}=2$
$\mathrm{T}=1 \mathrm{sec}$
21. $W=\frac{1}{2} k\left(x_{2}^{2}-x_{1}^{2}\right)$
22. Time period is independent of slope and hence $T=2 \pi\left(\frac{M}{2 K}\right)^{1 / 2}$
23. As here two masses are connected by two springs, this problem is equivalent to the oscillation of a reduced mass $m_{r}$ of a spring of effective spring constant.

$$
T=2 \pi \sqrt{\frac{m_{r}}{K_{\text {eff. }}}}
$$

Here $m_{r}=\frac{m_{1} m_{2}}{m_{1}+m_{2}}=\frac{m}{2} \Rightarrow K_{\text {eff. }}=K_{1}+K_{2}=2 K$
$\therefore n=\frac{1}{2 \pi} \sqrt{\frac{K_{\text {ef. }}}{m_{r}}}=\frac{1}{2 \pi} \sqrt{\frac{2 K}{m} \times 2}=\frac{1}{\pi} \sqrt{\frac{K}{m}}=\frac{1}{\pi} \sqrt{\frac{0.1}{0.1}}=\frac{1}{\pi} \mathrm{~Hz}$
24. When mass 700 gm is removed, the left out mass $(500+400) \mathrm{gm}$ oscillates with a period of 3 sec
$\therefore 3=t=2 \pi \sqrt{\frac{(500+400)}{k}}$
(i)

When 500 gm mass is also removed, the left out mass is 400 gm .
$\therefore t^{\prime}=2 \pi \sqrt{\frac{400}{k}}$
$\Rightarrow \frac{3}{t^{\prime}}=\sqrt{\frac{900}{400}} \Rightarrow t^{\prime}=2$ sec
25. (b) When the particle of mass $m$ at $O$ is pushed by $y$ in the direction of $A$ The spring $A$ will be compressed by $y$ while spring $B$ and $C$ will be stretched by $y^{\prime}=y \cos 45^{\circ}$. So that the total restoring force on the mass $m$ along $O A$.

$$
\begin{aligned}
& F_{n e t}=F_{A}+F_{B} \cos 45^{\circ}+F_{C} \cos 45^{\circ} \\
& =k y+2 k y^{\prime} \cos 45^{\circ}=k y+2 k\left(y \cos 45^{\circ}\right) \cos 45^{\circ}=2 k y
\end{aligned}
$$

Also $F_{\text {net }}=k^{\prime} y \Rightarrow k^{\prime} y=2 k y \Rightarrow k^{\prime}=2 k$


$$
T=2 \pi \sqrt{\frac{m}{k^{\prime}}}=2 \pi \sqrt{\frac{m}{2 k}}
$$

