Orbital and Escape Velocities

1. Orbital Velocity

a) The velocity required for a satellite to orbit round the earth very close to it is called orbital velocity (v_o).

b) Orbital Velocity $V_0 = \sqrt{\frac{GM}{R+h}} \cong \sqrt{\frac{GM}{R}} \cong \sqrt{gR}$

V_o = 7.92km/sec. for the earth bound satellites.

V_o = 1.7 kms^{-1} for moon bound satellites.

c) If the satellite is very close to the earth, then orbital angular velocity $\omega_0 = \sqrt{\frac{GM}{r^3}} \cong \sqrt{\frac{g}{R}}$ = 1.24×10^{-3} $rads^{-1}$.where r = R + h.

d) Angular momentum of the satellite is $L = m\sqrt{GM(R+h)}$

e) Orbital time period $T_0 = 2\pi\sqrt{\frac{r^3}{GM}}$

f) If the satellite is orbiting very close to the Earth, then $T_0 = 2\pi\sqrt{\frac{R}{g}} = 84\min.(Nearly)$ and

$T_0 = 2\pi\sqrt{\frac{3}{4\pi dg}}$

g) V_0 depends on mass of the planet and radius of the orbit.

h) If a body is taken from one orbit to another orbit , then

I. Radius of the orbit increases.

II. Orbital velocity and Orbital angular velocity decrease.

III. Time period of revolution increases.

IV. Angular momentum increases.

V. Kinetic energy decreases.

VI. Potential energy increases.

VII. Total energy increases.

i) A satellite of mass m orbiting close to the earth has kinetic energy and potential energy.

j) Kinetic energy of the satellite $= \frac{GMm}{2R} = \frac{mgR}{2}$

k) Potential energy of the satellite $= -\frac{GMm}{R}$

l) Total energy = K.E + P.E = $-\frac{GMm}{2R}$ (negative sign signifies that the body is bound to the earth)

m) If kinetic energy is E, then potential energy will be −2E and total energy will be −E.

n) The increase in gravitational potential energy of a body of mass 'm' taken to a height 'h' from the surface of the earth = $mgh\left(\frac{R}{R+h}\right) = \frac{GMmh}{R(R+h)}$.

2. Geo Stationary Satellite

a) An orbit in which the time period of revolution of a satellite is 24 hours is called geostationary orbit or parking orbit or synchronous orbit. It appears stationary with respect to the earth.

b) Radius of the geo–stationary orbit is approximately 42,400 km. Speed of geo–stationary satellite in it is 3.1 kms^{-1}.

c) The relative velocity of a geostationary satellite with respect to the earth is zero.

d) Height of the parking orbit is 36,000 km approximately from the surface of earth

$$h = \left(\frac{GM}{4\pi^2}T^2\right)^{1/3} - R$$

e) Geo stationary satellite orbits above the equator in the equatorial plane.

f) Gravitational mass cannot be determined in a geo-stationary satellite ($\because g = 0$)

g) Satellite is launched from the equatorial region, from west to east direction, with a velocity same as that of the revolution of the earth.

h) Geostationary satellites are used

1. To study the upper layers of atmosphere.
2. To forecast the changes in the atmosphere.

3. To know the shape and size of the earth.
4. To identify the minerals and natural resources present inside and on the surface of the earth.
5. To transmit the T.V. programs to distant places.
6. To undertake extensive research work on the planets, satellites and comets etc., which are present in space.

3. Escape velocity

a) A body is taken to a height nR above the surface of the Earth. The change in PE is given by

Change in PE = $\frac{GMm}{R} - \frac{GMm}{(R+nR)} = \frac{GMm}{R}\left(\frac{n}{n+1}\right) = mgR\left(\frac{n}{n+1}\right)$.

b) The height to which the body reaches, if it is projected with a velocity v, $h = \frac{v^2R}{2gR - v^2}$

c) The work done in lifting a body of mass m from the surface of the earth to a height h is given by $W = \frac{mgh}{\left(1+\frac{h}{R}\right)}$.

d) Velocity with which a body is projected so that it reaches a height 'h' $v = \sqrt{\frac{2Rgh}{R+h}}$

e) The escape velocity of a body on earth or on any planet is $V_e = \sqrt{2gR}$ or $V_e = \sqrt{\frac{2GM}{R}}$.

f) It depends upon mass M of the earth or planet and radius R of the earth or planet.

It is independent of mass of the body and angle of projection. Its value on earth surface is $11.2 Kms^{-1}$. V_e on moon = $2.5\ kms^{-1}$

g) If r. m. s. velocity of gas molecules is equal or greater than escape velocity, then there will be no atmosphere.

h) Orbital velocity and escape velocity are related as $V_e = \sqrt{2}\, v_0$

i) When a body is projected with escape velocity its total energy is zero.

4. Both the escape velocity and the orbital velocity are independent of the mass of the body.