## Simple Pendulum

1. Pendulum clock shows correct time at $\mathrm{O}^{\circ} \mathrm{C}$. At a higher temperature the clock
1) Loses time
2) Gains time
3) Neither gains nor loses time
4) All the above are possible depending on the co-efficient of linear expansion of the pendulum
2. A second's pendulum is taken from the surface of the earth to that of the moon. In order to maintain the period constant
1) Length of the pendulum has to be decreased
2) Length of the pendulum has to be increased
3) Amplitude of the pendulum has to be increased
4) Amplitude of the pendulum has to be decreased
3. If iron sphere is replaced by wooden sphere of same mass, time period
1) Increases
2) Decreases
3) Remains same
4) None of the above
4. The bob of a simple pendulum of period $T$ is given a negative charge. If it is allowed to oscillate with an identical charge at the point of suspension, the new time period will be
1) Equal $T$
2) More than $T$
3) Less than $T$
4) Infinite.
5. The time period of a pendulum in stationary lift is ' T ', if lift starts accelerating in the downward direction, the time period will
1) Increase
2) Decrease
3) No change
4) Nothing certain
6. Identify the correct statement among the following.
1) The greater the mass of the pendulum bob, the shorter is its frequency of oscillation.
2) A simple pendulum with a bob of mass ' m ' swing with angular amplitude of $40^{\circ}$. When its angular amplitude is $20^{\circ}$ the tension in the string is less than $\mathrm{Mg} \cos 20^{\circ}$.
3) As the length of a simple pendulum is increased, the maximum velocity of its bob during its oscillation will also increase.
4) The fractional change in the time period of a pendulum on changing the temperature is in independent of the length of the pendulum.
7. The restoring force which brings the bob towards the mean position is equal to
1) Weight of bob
2) Radial component
3) Tangential component of weight of bob
4) Resultant of radial and tangential component of weight of bob
8. To show that a simple pendulum is in a simple harmonic motion it is necessary to assume that
1) The length of the pendulum is small.
2) The mass of the pendulum is small.
3) The amplitude of oscillation is small.
4) The acceleration due to gravity is small.
9. A simple pendulum hanging freely and at rest is vertical because in that position
1) K.E is zero
2) K.E minimum
3) P.E. is zero
4) P.E. minimum
10. A second's pendulum is taken from the surface of the earth to that of the moon in order to maintain the period constant
1) The length of the pendulum has to be decreased
2) The length of the pendulum has to be increased
3) The amplitude of the pendulum should be decreased
4) No change is required
11. (A): A hollow sphere is filled with water through a small hole in it. It is suspended by a long thread and as the water flows out of the hole at the bottom, it is that the period of oscillation first increases and then decreases.
$(\mathrm{R}):$ The time period depends on the effective length of the pendulum.
(1) Both A and R are true and R is the correct explanation of A.
(2) Both $A$ and $R$ are true but $R$ is not the correct explanation of $A$.
(3) $A$ is true but $R$ is false.
(4) A is false but R is true.
12. (A): The bob of a pendulum is immersed in a non viscous liquid (denser than water) completely. Time period of pendulum increases.
$(\mathrm{R})$ : Effective acceleration due to gravity increases.
1) $A$ and $R$ are true and $R$ is correct explanation for $R$.
2) $A$ is true but $R$ is not correct explanation for $A$.
3) A is true $R$ is false.
4) $A$ is false but $R$ is true.
13. (A): The metal bob of pendulum is positively charged and made to oscillate over a positively charged plate. Its time period increases.
( R : Time period in the given case is given by $T=2 \pi \sqrt{\frac{1}{\mathrm{~g}-\frac{\mathrm{qE}}{\mathrm{m}}}}$.
1) Both $A$ and $R$ are true and $R$ is correct explanation for $A$.
2) Both $A$ and $R$ are true but $R$ is not correct explanation for $A$.
3) A is true but $R$ is false.
4) A is false but $R$ is true.
14. A simple pendulum is under acceleration. The time period of a stationary pendulum is $\mathbf{T}$ and the modified period $T^{\mathbf{1}}$. Match the ratio $\mathbf{T}^{1 /} / \mathbf{T}$ for various situations.
$\mathrm{T}^{1 / T}$
(a) $\sqrt{\frac{g}{g-a}}$
(b) $\sqrt{\frac{g}{g}}$
(f) Vehicle moving horizontally with acceleration
(c) $\infty$
(g) Lift with acceleration moving up
(d) $\sqrt{\frac{g}{\sqrt{g^{2}+a^{2}}}}$
(h) Lift with acceleration moving down
1) $a-h, b-g, c-e, d-f$
2) $a-g, b-e, c-f, d-h$
3) a-h, b-f, c-e, d-g
4) a-g, b-h, c-e, d-f
15. Pendulum beating seconds at the equator ( $\mathrm{g}=\mathbf{9 7 8} \mathrm{cm} / \mathrm{sec}^{2}$ ) is taken to Antarctica ( $\mathrm{g}=\mathbf{9 8 3} \mathrm{cm} / \mathrm{sec}^{2}$ ), its length is to be increased by
1) 0.1 cm
2) 0.2 cm
3) 0.4 cm
4) 0.5 cm
16. The mass and diameter of a planet are twice that of the earth. What will be the time period of oscillation of a pendulum on this planet, if it is a second's pendulum on earth?
1) $\sqrt{2} \mathrm{Sec}$
2) 2 sec
3) $\frac{1}{\sqrt{2}} \mathrm{sec}$
4) $2 \sqrt{2} \mathrm{sec}$
17. If a simple pendulum of length ' $I$ ' is about to start to make SHM, when it is brought a position so that its length has maximum angular displacement ' $\theta$ '. If the mass of the bob is ' $\mathbf{m}$ ', then the maximum K.E at its mean position is
1) $\frac{1}{2} \mathrm{~m}\left(\frac{l}{g}\right)$
2) $\frac{1}{2} \mathrm{mgl} \sin \theta$
3) 2 mg 1
4) $\mathrm{mgl}(1-\cos \theta)$

18. The time period of simple pendulum is ' $T$ '. When the length increases by $10 \mathbf{c m}$, its period is $T_{1}$. When the length is decreased by 10 cm , its period is $T_{2}$. Then the relation between $T, T_{1}$ and $T_{2}$ is
1) $\frac{2}{\mathrm{~T}^{2}}=\frac{1}{\mathrm{~T}_{1}^{2}}+\frac{1}{\mathrm{~T}_{2}^{2}}$
2) $\frac{2}{\mathrm{~T}^{2}}=\frac{1}{\mathrm{~T}_{1}^{2}}-\frac{1}{\mathrm{~T}_{2}^{2}}$
3) $2 T^{2}=T_{1}{ }^{2}+T_{2}{ }^{2}$
4) $2 T^{2}=T_{1}{ }^{2}-T_{2}{ }^{2}$
19. A simple pendulum has a time period ' $T$ ' in vacuum. Its time period when it is completely immersed in a liquid of density one - eighth of the density of the material of the bob is
1) $\sqrt{\frac{7}{8}} \mathrm{~T}$
2) $\sqrt{\frac{5}{8}} \mathrm{~T}$
3) $\sqrt{\frac{3}{8}} \mathrm{~T}$
4) $\sqrt{\frac{8}{7}} \mathrm{~T}$
20. The bob of a simple pendulum is displaced from its equilibrium position $O$ to a position $Q$ which is at height $h$ above $O$ and the bob is then released. Assuming the mass of the bob to be $\boldsymbol{m}$ and time period of oscillations to be 2.0 sec , the tension in the string when the bob passes through $O$ is
1) $m(g+\pi \sqrt{2 g h})$
2) $m\left(g+\sqrt{\pi^{2} g h}\right)$
3) $m\left(g+\sqrt{\frac{\pi^{2}}{2} g h}\right)$
4) $m\left(g+\sqrt{\frac{\pi^{2}}{3} g h}\right)$

21. A cylindrical piston of mass $M$ slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be
1) $T=2 \pi \sqrt{\left(\frac{M h}{P A}\right)}$
2) $T=2 \pi \sqrt{\left(\frac{M A}{P h}\right)}$
3) $T=2 \pi \sqrt{\left(\frac{M}{P A h}\right)}$
4) $T=2 \pi \sqrt{M P h A}$

22. A sphere of radius $r$ is kept on a concave mirror of radius of curvature $\boldsymbol{R}$. The arrangement is kept on a horizontal table (the surface of concave mirror is frictionless and sliding not rolling). If the sphere is displaced from its equilibrium position and left, then it executes S.H.M. The period of oscillation will be
1) $2 \pi \sqrt{\left(\frac{(R-r) 1.4}{g}\right)}$
2) $2 \pi \sqrt{\left(\frac{R-r}{g}\right)}$
3) $2 \pi \sqrt{\left(\frac{r R}{a}\right)}$
4) $2 \pi \sqrt{\left(\frac{R}{g r}\right)}$
23. A $\boldsymbol{U}$ tube of uniform bore of cross-sectional area $\boldsymbol{A}$ has been set up vertically with open ends facing up. Now $m \mathrm{gm}$ of a liquid of density $\boldsymbol{d}$ is poured into it. The column of liquid in this tube will oscillate with a period $T$ such that
1) $T=2 \pi \sqrt{\frac{M}{g}}$
2) $T=2 \pi \sqrt{\frac{M A}{g d}}$
3) $T=2 \pi \sqrt{\frac{M}{g d A}}$
4) $T=2 \pi \sqrt{\frac{M}{2 \mathrm{Adg}}}$
24. A simple pendulum is hanging from a peg inserted in a vertical wall. Its bob is stretched in horizontal position from the wall and is left free to move. The bob hits on the wall the coefficient of restitution is $\frac{2}{\sqrt{5}}$. After how many collisions the amplitude of vibration will become less than $60^{\circ}$
1) 6
2) 3
3) 5
4) 4
25. A simple pendulum has time period $\boldsymbol{T}_{1}$. The point of suspension is now moved upward according to equation $y=k t^{2}$ where $k=1 \mathrm{~m} / \mathrm{sec}^{2}$. If new time period is $\boldsymbol{T}_{\mathbf{2}}$ then ratio $\frac{T_{1}^{2}}{T_{2}^{2}}$ will be
1) $2 / 3$
2) $5 / 6$
3) $6 / 5$
4) $3 / 2$

Key

1) 1
2) 1
3) 1
4) 1
5) 4
6) 3
7) 3
8) 4
9) 1
10) 1
11) 1
12)2
12) 1
13) 1
14) 4
15) 4
16) 4
17) 3
18) 4
19) 1
20) 1
21) 2
22) 4
24)2
23) 3

## Hints

15. $I=\frac{g}{\pi^{2}}=\frac{983-978}{10}$

$$
=\frac{5}{10}=0.5 \mathrm{~cm}
$$

16. $T \propto \frac{1}{\sqrt{g}}$

$$
g=\frac{m}{R^{2}}
$$

$$
g^{1} \propto \frac{G .2 m}{G R^{2}}=\frac{G M}{2 R^{2}}
$$

$$
g^{1} \propto g / 2
$$

$$
\frac{T_{1}}{T_{2}}=\sqrt{\frac{g^{1}}{g}}
$$

$$
\frac{2}{T_{2}}=\sqrt{\frac{g / 2}{g}}=\frac{1}{\sqrt{2}}
$$

$$
\mathrm{T}_{2}=2 \sqrt{2} \mathrm{sec}
$$

17. $m g l(1-\cos \theta)$
18. $T \propto \sqrt{l}$
$(l+10) \propto T_{1}^{2}$
$(l-10) \propto T_{2}^{2}$

$$
2 T^{2}=T_{1}^{2}+T_{2}^{2}
$$

19. $T \propto \frac{1}{\sqrt{g}}$

$$
\begin{aligned}
& \frac{T}{T_{2}}=\sqrt{\frac{g\left[1-\frac{d_{B}}{d_{B}}\right]}{g}} \\
& \frac{T}{T_{2}}=\sqrt{1-\frac{d_{B}}{8 \cdot d_{B}}}=\sqrt{\frac{7}{8}}\left(\therefore d_{l}=\frac{d_{B}}{8}\right) \\
& T_{2}=\sqrt{\frac{8}{7} T}
\end{aligned}
$$

20. Tension in the string when bob passes through lowest point

$$
T=m g+\frac{m v^{2}}{r}=m g+m v \omega
$$

$$
(\because v=r \omega)
$$

Putting $v=\sqrt{2 g h}$ and $\omega=\frac{2 \pi}{T}=\frac{2 \pi}{2}=\pi$
We get $T=m(g+\pi \sqrt{2 g h})$
21. Let the piston be displaced through distance $x$ towards left, then volume decreases, pressure increases. If $\Delta P$ is increase in pressure and $\Delta V$ is decrease in volume, then considering the process to take place gradually (i.e. isothermal)

$$
\begin{aligned}
& P_{1} V_{1}=P_{2} V_{2} \Rightarrow P V=(P+\Delta P)(V-\Delta V) \\
& \Rightarrow P V=P V+\Delta P V-P \Delta V-\Delta P \Delta V \\
& \Rightarrow \Delta P \cdot V-P \cdot \Delta V=0 \quad \text { (Neglecting } \Delta P . \Delta V)
\end{aligned}
$$


$\Delta P(A h)=P(A x) \Rightarrow \Delta P=\frac{P \cdot x}{h}$
This excess pressure is responsible for providing the restoring force $(F)$ to the piston of mass $M$.

Hence $F=\Delta P . A=\frac{P A x}{h}$
Comparing it with $|F|=k x \Rightarrow k=M \omega^{2}=\frac{P A}{h}$

$$
\Rightarrow \omega=\sqrt{\frac{P A}{M h}} \Rightarrow T=2 \pi \sqrt{\frac{M h}{P A}}
$$

22. Tangential acceleration, $a_{t}=-g \sin \theta=-g \theta$
$a_{t}=-g \frac{x}{(R-r)}$
Motion is S.H.M., with time period

$T=2 \pi \sqrt{\frac{\text { displaceme } \mathrm{nt}}{\text { acceleration }}}=2 \pi \sqrt{\frac{x}{\frac{g X}{(R-r)}}}=2 \pi \sqrt{\frac{R-r}{g}}$
23. If the level of liquid is depressed by $y \mathrm{~cm}$ on one side, then the level of liquid in column $P$ is $2 y \mathrm{~cm}$ higher than $B$ as shown.


The weight of extra liquid on the side $P=2$ Aydg
This is the restoring force on mass $M$.
$\therefore$ Restoring acceleration $=\frac{-2 \text { Aydg }}{M}$
This is the condition of SHM i.e. $a \propto-y$.

Hence time period $T=2 \pi \sqrt{\frac{\text { Displaceme } \mathrm{nt}}{\mid \text { Acceleration } \mid}}$

$$
=2 \pi \sqrt{\frac{y}{\frac{2 A y d g}{M}}} \Rightarrow T=2 \pi \sqrt{\frac{M}{2 A d g}}
$$

24. From the relation of restitution $\frac{h_{n}}{h_{0}}=e^{2 n}$ and
$h_{n}=h_{0}\left(1-\cos 60^{\circ}\right) \Rightarrow \frac{h_{n}}{h_{0}}=1-\cos 60^{\circ}=\left(\frac{2}{\sqrt{5}}\right)^{2 n}$
$\Rightarrow 1-\frac{1}{2}=\left(\frac{4}{5}\right)^{n} \Rightarrow \frac{1}{2}=\left(\frac{4}{5}\right)^{n}$
Taking log of both sides we get

$$
\begin{aligned}
& \log 1-\log 2=n(\log 4-\log 5) \\
& 0-0.3010=n(0.6020-0.6990) \\
& -0.3010=-n \times 0.097 \Rightarrow n=\frac{0.3010}{0.097}=3.1 \approx 3
\end{aligned}
$$

25. $y=K t^{2} \Rightarrow \frac{d^{2} y}{d t^{2}}=a_{y}=2 K=2 \times 1=2 \mathrm{~m} / \mathrm{s}^{2}\left(\because K=1 \mathrm{~m} / \mathrm{s}^{2}\right)$

Now, $T_{1}=2 \pi \sqrt{\frac{l}{g}}$ and $T_{2}=2 \pi \sqrt{\frac{l}{\left(g+a_{y}\right)}}$
Dividing, $\frac{T_{1}}{T_{2}}=\sqrt{\frac{g+a_{\mathrm{x}}}{g}} \Rightarrow \sqrt{\frac{6}{5}} \Rightarrow \frac{T_{1}^{2}}{T_{2}^{2}}=\frac{6}{5}$

