Simple Pendulum

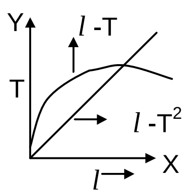
1. The period of oscillation of a simple pendulum is independent of amplitude (for small values only), length being constant.
2. At constant length, the period of oscillation of a simple pendulum is independent of size, shape or material of the bob.

   Time period of a simple pendulum (T) = $2\pi\sqrt{\frac{L}{g}}$. Where

   l = length of the simple pendulum

   g = acceleration due to gravity at a place.
3. Tension in the string of simple pendulum
   a. T_{min} = mg Cos θ (when bob is at extreme position)
   b. T = mg (3 - 2 Cosθ) (When bob is at any position)

   Where θ is any angular amplitude.
4. I - T^2 graph of a simple pendulum is straight line passing through origin.
5. l-T graph of a simple pendulum is parabola.
6. At the point of intersection of l-T graph and l–T^2 graph of a simple pendulum.



   i) T = 1 second

   ii) n = 1 Hz.

   iii) $l = \frac{g}{4\pi^2} \cong 25\text{cm}$ On the surface of the earth
7. If $L = \infty$ (infinity). $T = 2\pi\sqrt{\frac{R}{g}} = $ 84.5 min.
8. If $L = R$, $T = 2\pi\sqrt{\frac{R}{2g}} = \frac{84.5}{\sqrt{2}}$ min
9. If L is very small compared to Radius of the earth, $T = 2\pi\sqrt{\frac{\ell}{g}}$

10. Restoring Force on the bob of the pendulum is $F = mg\sin\theta$.

11. Seconds pendulum

i) The simple pendulum whose time period equal to 2 seconds is called seconds pendulum.

ii) The length at place where g = 9.8 m/s^2 is 100 cm.

iii) Since T = 2 sec, L = $\frac{g}{\pi^2}$

iv) For two places, change in length = $\frac{g_1 \sim g_2}{\pi^2}$

Application

i) When the elevator is going up with an acceleration a, then its time period is given by T = $2\pi\sqrt{\frac{L}{g+a}}$.

ii) When the elevator is moving down with an acceleration a, then its time period is given by T = $2\pi\sqrt{\frac{L}{g-a}}$.

iii) When the elevator is at rest or moving up or down with constant velocity the time period is given by T = 2$T = 2\pi\sqrt{\frac{L}{g}}$.

iv) When the elevator is moving down with an acceleration (-a) then its time period is given by $T = 2\pi\sqrt{\frac{L}{g+a}}$.

v) In case of downward accelerated motion is a > g the pendulum turns upside and oscillates about the highest point with T = $2\pi\sqrt{\frac{L}{a-g}}$.

vi) If a simple pendulum of length 'L' suspended in a car that is travelling with a constant speed around a circle of radius 'r', then its time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}}.$$

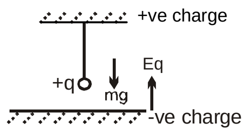
vii) If a simple pendulum of length 'L' suspended in car moving horizontally with acceleration 'a' is given by $T = 2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$.

The equilibrium position is inclined to the vertical by an angle 'θ'. Where $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ opposite to the acceleration.

viii) If the bob of a simple pendulum is given a charge 'q' and is arranged in an electric field of intensity 'E' to oscillate.

a) Opposite to g, → Electric force E_q will be opposite to the force mg. Hence $g^1 = g - \frac{Eq}{m}$ .Then $T_1 = 2\pi\sqrt{\frac{l}{g-\frac{Eq}{m}}}$. So time period increases.

b) In the direction of g → Electric force Eq will be in the direction of force mg. Hence $g^1 = g + \frac{Eq}{m}$ then $T_1 = 2\pi\sqrt{\frac{l}{g+\frac{Eq}{m}}}$ so time period decreases.



c) Perpendicular to g → Electric force E_q will be perpendicular to the force mg. Hence $g^1 = \sqrt{g^2+\left(\frac{Eq}{m}\right)^2}$ Then $T_1 = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{Eq}{m}\right)^2}}}$. So time period decreases.

ix) If a simple pendulum of length L is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination 'θ'. Then the time period of oscillations is given by $T = 2\pi\sqrt{\frac{L}{g\cos\theta}}$ since the effective acceleration changes from g to g cosθ.

x) A simple pendulum fitted with a metallic bob of density d_s has a time period T. When it is made to oscillate in a liquid of density d_1 then its time period increases. $T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{d_l}{d_s}\right)}}$.

12. Time period of Torsion pendulum $T = 2\pi\sqrt{\frac{I}{C}}$ I = moment of Inertia about the suspension wire C = couple per unit twist.

13. When a hole is drilled along the diameter of the earth and if a body is dropped in it, it moves to and from about the centre of the earth and is in S.H M. with a time period of

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes.}$$