Motion under Gravity

1. Freely falling body

a) The equations of motion

i) $V = u + at$ ii) $s = ut + \frac{1}{2}at^2$

$V = gt$ $h = \frac{1}{2}gt^2$

$V \propto t$ $h \propto t^2$

$\frac{V_1}{V_2} = \frac{t_1}{t_2}$ $\frac{h_1}{h_2} = \frac{t_1^2}{t_2^2}$

iii) $V^2 = u^2 + 2as$ iv) $s_n = u + \frac{a}{2}(2n-1)$

$V^2 = 2gh$ $h_n = \frac{g}{2}(2n-1)$

$V^2 \propto h$ $h_n \propto (2n-1)$

$\frac{V_1}{V_2} = \frac{h_1}{h_2}$

b) The average velocity during fall (V) = $\sqrt{\frac{gh}{2}}$

c) The ratio of distance traveled in 1^{st}, 2^{nd}, 3^{rd}….. n seconds is 1:3:5……(2n-1)

d) The ratio of distances traveled in first, first two, first three seconds…… is 1:4:9…..n^2

e) The ratio of time taken to travel first, 2^{nd}, 3^{rd}……n^{th} unit of distances is

$1 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2} \ldots\ldots \left(\sqrt{n}-\sqrt{n-1}\right)$

f) The ratio of times taken to travel first, first two, first three …first n units of distances is $1 : \sqrt{2} : \sqrt{3} : \ldots\ldots\ldots \sqrt{n}$

g) If x is the distance traveled in the n^{th} second, then the distance traveled in the $(n+1)^{th}$ second is $\left(\frac{2n+1}{2n-1}\right)x$ (or) x + g.

h) If x is the distance traveled in the n^{th} second, then distance traveled in the $(n-1)^{th}$ second is $\left(\frac{2n-3}{2n-1}\right)x$ (or) x - g

i) The ratio of distances covered in the nth second and the distance traveled in n seconds is $\frac{s_n}{s} = \frac{2}{n} - \frac{1}{n^2} = \frac{2n-1}{n^2}$

j) $S_{n+1} - s_n = g$

k) If a body travels $\frac{1}{n}$th of the total distance in the last second the total time of fall

$T = \left[n + \sqrt{n(n-1)}\right]$

l) If a particle takes x seconds less and acquires a velocity y ms^{-1} more at one place than at another in falling through the same distance. If g_1 and g_2 are accelerations due to gravity at these two places, then x: y is $\left(1/\sqrt{g_1 g_2}\right)$.

m) The acceleration of a body in a medium is given by $g^I = g\left(\frac{1-d_m}{d_b}\right) = g\left(1-\frac{d_m}{d_b}\right)$

Where d_m = density of the medium and d_b = density of the body

n) If a body is dropped into a well of depth h the time taken to hear the sound from start (v is the velocity of sound) T is given by $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

2. Body thrown vertically upwards

a) The equations of motion

a) v = u + at b) $s = ut + ½ at^2$

v = u – gt $h = ut – ½ gt^2$

c) $V^2 = u^2 + 2as$ d) $s_n = u + \frac{a}{2}(2n-1)$

$u = \sqrt{2gh}$ $h_n = u - \frac{g}{2}(2n-1)$

b) Maximum height reached = H = $\frac{u^2}{2g}$

c) Time of ascent = Time of descent = $\frac{u}{g}$

Time of flight $= \frac{2u}{g}$

d) Maximum height H = $\frac{1}{2g}\left(\frac{gT}{2}\right)^2 = \frac{gT^2}{8}$

e) The velocity of the body at the half of the maximum height is $\sqrt{gh}$ (or) $\sqrt{\frac{u^2}{2}}$

f) A body projected vertically up from the top of a tower of height h reaches the ground in a time t , then $h = -ut + \frac{1}{2}gt^2$ and $h = \frac{v^2 - u^2}{2g}$

g) A body is projected up with a velocity u and another body is also projected up from the same point with same velocity but after t sec. Then they will meet after a time $T = \frac{u}{g} + \frac{t}{2}$

h) A body projected up from the top of a tower with a velocity u reaches the ground in a time t_1. Another body projected down with same velocity reaches the ground in time t_2

i) The time difference $(t_1 - t_2) = \frac{2u}{g}$

ii) Time take by the freely falling body to reach the ground is $\sqrt{t_1 t_2}$

iii) Height of the tower is $h = \frac{1}{2}gt_1t_2$

iv) Velocity of projection is $u = \frac{g}{2}(t_1 - t_2)$

i) If air resistance is considered, time of ascent < time of descent.