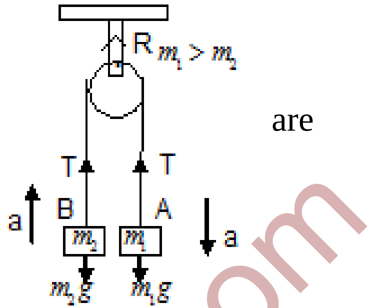
Dynamics-Connected Bodies

1. Atwood's machine

a) For body A, $m_1 g - T = m_1 a$

For body B, $T - m_2 g = m_2 a$



$\therefore$ Acceleration and tension of the system respectively are

$a = \dfrac{(m_1 - m_2)g}{(m_1 + m_2)}$ And $T = \dfrac{2m_1 m_2 g}{(m_1 + m_2)}$

b) $a_{CM} = \dfrac{(m_1 a_1 - m_2 a_2)}{(m_1 + m_2)} = \dfrac{(m_1 - m_2)a}{(m_1 + m_2)} = \dfrac{(m_1 - m_2)}{(m_1 + m_2)}\dfrac{(m_1 - m_2)}{(m_1 + m_2)} g$

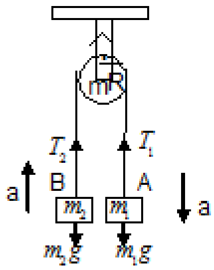
$\therefore a = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

c) The reaction at the pulley, R = 2T $\Rightarrow R = \dfrac{4m_1 m_2 g}{(m_1 + m_2)}$

2. If the mass of the pulley is also taken into consideration, then

For body A, $m_1 g - T_1 = m_1 a$ --- (1)

For body B, $T_2 - m_2 g = m_2 a$ --- (2)



For the pulley, $(T_1 - T_2)R = I\alpha$, where R is the radius of the pulley.

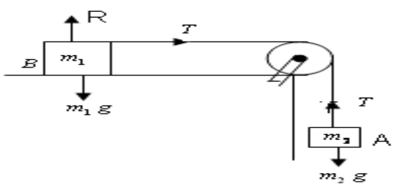
$(T_1 - T_2)R = \dfrac{1}{2}MR^2 \dfrac{a}{R}$

$T_1 - T_2 = \dfrac{1}{2}Ma$ --- (3)

From equations (1), (2) and (3)

$a = \dfrac{(m_1 - m_2)g}{\left(m_1 + m_2 + \dfrac{m}{2}\right)}$, $T_1 = \dfrac{m_1\left[2m_2 + \dfrac{M}{2}\right]g}{m_1 + m_2 + \dfrac{M}{2}}$ and $T_2 = \dfrac{m_2\left[2m_1 + \dfrac{M}{2}\right]g}{m_1 + m_2 + \dfrac{M}{2}}$

3. One object having horizontal motion and other having vertical motion:



a) For the body at A, $m_2 g - T = m_2 a$

For the body at B, $T = m_1 a$

$$\therefore a = \frac{m_2 g}{(m_1 + m_2)} \quad \text{and} \quad T = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

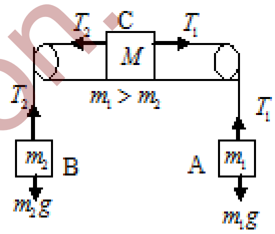
Thrust on the pulley is $\sqrt{2}T$

b) If the coefficient of friction between table and the mass is 'μ' then

For the body at A, $m_2 g - T = m_2 a$

For the body at B, $T - \mu m_1 g - = m_1 a$

$$a = \frac{(m_2 - \mu m_1) g}{(m_1 + m_2)} \quad \text{And} \quad T = \frac{m_1 m_2 (1+\mu) g}{(m_1 + m_2)}$$



4. For the body at A, $m_1 g - T_1 = m_1 a$

For the body at A, $T_2 - m_2 g = m_2 a$

For the body at C, $T_1 - T_2 = Ma$

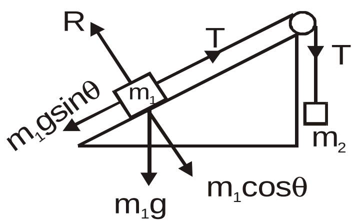
$$a = \frac{(m_1 - m_2) g}{(m_1 + m_2 + M)} \quad \text{and}$$

$$T_1 = \frac{m_1 [2m_2 + M] g}{m_1 + m_2 + M} \quad \text{and} \quad T_2 = \frac{m_2 [2m_1 + M] g}{m_1 + m_2 + M}$$

5. Two masses m_1 and m_2 connected by a string pass over a pulley. m_2 is suspended and $m_1$slides up over a frictionless inclined plane of angle θ

$T - m_1 g \, Sin\,\alpha = m_1 a$

$m_2 g - T_1 = m_2 a$



Acceleration, $a = \dfrac{(m_2 - m_1 \sin\theta) g}{m_1 + m_2}$

and

Tension in the string $T = m_2 g - m_2 a = \dfrac{m_1 m_2 [1 + \sin\theta] g}{(m_1 + m_2)}$

6. $m_1 g \, Sin\,\alpha - T = m_1 a$

$T - m_2 g \, Sin\,\beta = m_2 a$

Tension (T) $= \dfrac{m_1 m_2 (\sin\alpha + \sin\beta) g}{m_1 + m_2}$

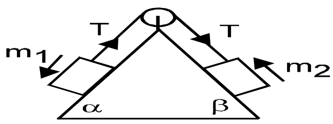
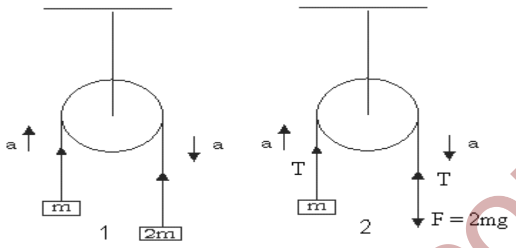


Fig (iv)

Acceleration (a) = $\dfrac{g(m_1 \sin\alpha - m_2 \sin\beta)}{m_1 + m_2}$

Force on the pulley $F = 2T\,Cos\left(\dfrac{90-\theta}{2}\right)$

7. The acceleration in the following case



a) $a = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\dfrac{m}{3m}\right)g = \dfrac{g}{3}$

b) T= pulling force F = 2mg

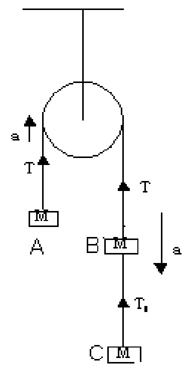
T – mg = ma'

mg = ma'

a' = g

8. Consider the following system

a) $a = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\dfrac{2M - M}{2M + M}\right)g = \dfrac{g}{3}$



b) Tension in the string AB is

T – Mg = Ma

$T = Mg + Ma = \dfrac{4Mg}{3}$

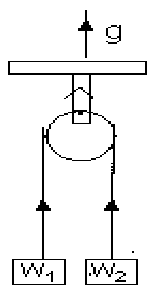
c) Tension in the string BC is

$Mg - T_1 = Ma$

$T_1 = Mg - Ma$

Or $T_1 = \dfrac{2Mg}{3}$

9. Two weights w_1 and w_2 are suspended as shown. When the pulley is pulled up with an acceleration g, the tension in the string is

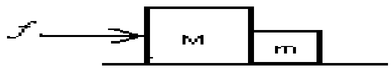


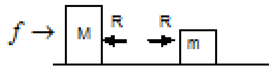
$T = \dfrac{2m_1m_2}{(m_1 + m_2)}(g + g)$

Or $T = \dfrac{4m_1m_2g}{(m_1 + m_2)} = \dfrac{4w_1w_2}{(w_1 + w_2)}$

10. Bodies in contact:

a. Consider two bodies m and M which are in contact and placed on a horizontal smooth surface. Let a force f is applied on the system as shown. Let R be the contact force between the two bodies

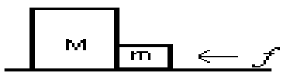


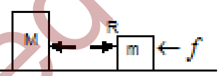


1) $a = \dfrac{f}{M+m}$

2) $f - R = Ma$ and $R = ma$

b.

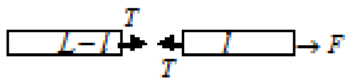




1) $a = \dfrac{f}{M+m}$

2) $f - R = ma$ and $R = Ma$

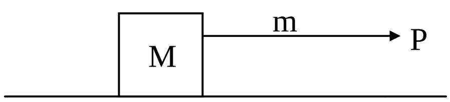
11. A uniform rope of length L is placed on a horizontal smooth surface and pulled with a force F at one of its end. Let m be the mass per unit length of the rope. The tension in the rope at a distance l force the end where force is applied is given by



a) $a = \dfrac{F}{mL}$

b) $T = \left(\dfrac{L-l}{L}\right)F$

12. A block of mass M is pulled by a rope of mass m by a force P on a smooth horizontal plane.

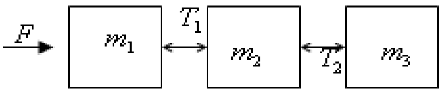


a) Acceleration of the block $a = \frac{p}{M+m}$

b) Force exerted by the rope on the block

$$F = \frac{Mp}{(M+m)}$$

13. Consider the following system



For the 1st body, $F - T_1 = m_1 a$

For the second body, $T_1 - T_2 = m_2 a$

For the third body, $T_2 = m_3 a$

$\therefore$ Acceleration of the system

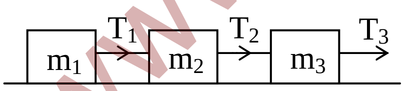
$$a = \frac{F}{m_1 + m_2 + m_3} \quad \text{And} \quad T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

$$T_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

If the force (F) acts on m_3, then

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)} \text{ And } T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

14. Masses m_1, m_2, m_3 are inter connected by light string and are pulled with a string with tension T_3 on a smooth table.



a) Acceleration of the system

$$a = \frac{T_3}{(m_1 + m_2 + m_3)}$$

b) Tension in the string

$$T_1 = m_1 a = \frac{m_1 T_3}{m_1 + m_2 + m_3}$$

$$T_2 = (m_1 + m_2)a = \frac{(m_1 + m_2)T_3}{m_1 + m_2 + m_3}$$

$T_3 = (m_1 + m_2 + m_3)\ a$