Equations

1. Any motion repeated at regular intervals of time is called periodic motion.
   Eg: Earth revolution around itself, simple pendulum.
2. If there is retracement of the path, it is called harmonic motion. Ex: Simple pendulum.
3. Every harmonic is periodic. But every periodic is not harmonic.
4. Angular SHM: Simple pendulum, Sting under vibrations, vibrations of a tuning fork etc.
5. Linear SHM: Loaded spring, a body dropped in a tunnel along the diameter of the earth, a liquid in a U-tube etc.
6. In SHM acceleration is directly proportional to the displacement of the particle from the fixed point, and the acceleration is always directed towards the fixed point in the path of the body.

   $a \;\alpha\; -x$ (Or) $a = -kx$

   Also $F \alpha - x$ (or) $F = -k\,x$

   Acceleration (a) $= \frac{F}{m} = \frac{-k}{m} x$.

   $a = -\omega^2 x$ (Or) $\frac{d^2x}{dt^2} + \omega^2 x = 0$ Where $\omega = \sqrt{\frac{k}{m}}$

   This is the differential equation of a body executing SHM.
7. For a body executing SHM,

   Variables: Displacement, Velocity, KE, PE, acceleration.

   Constants: TE, Time Period, Frequency, Amplitude, angular Velocity.
8. **Phase**

   a. Phase represents the state of vibration of a vibrating body from the mean position expressed in degrees (or) radians.

b. If two particles are in phase, the phase difference may be $0, 2\pi, 4\pi, 6\pi, ...2n\pi$ where n is an integer.

c. Phase difference = $\frac{2\pi}{\lambda} \times$ path difference.

$\phi = \frac{2\pi}{\lambda} x$ Where λ is the wavelength

d. Phase difference = $\frac{2\pi}{T} \times$ Time difference

$\phi = \frac{2\pi}{T} t$ Where T is the time period

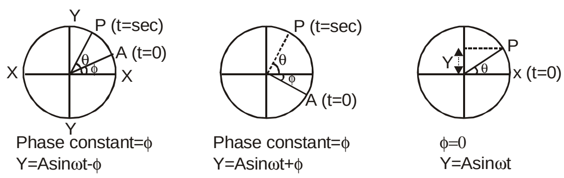
e. If at two instants of time t_1 and t_2 for a vibrating body. The time period is

$\phi = \omega(t_2 - t_1) = \frac{2\pi}{T}(t_2 - t_1)$.

And for two time periods $\phi = 2\pi t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$.

9. Amplitude: The maximum displacement of a vibrating particle from its mean position is called amplitude (A). It is a vector quantity. If a_1 and a_2 are the amplitudes of two SHMs with a phase difference ϕ, the resultant amplitude is given by $a^2 = a_1^2 + a_2^2 + 2a_1, a_2 \cos\phi$.

10. Representation of S. H. M.: When a particle is executing periodic motion along a circular path, the foot of the perpendicular drawn from the instantaneous position of a particle on any diameter executes simple harmonic motion.



The simple harmonic motion is represented as

Y = A sin (w t +ϕ)

Y = instantaneous displacement

A = Amplitude

Wt + ϕ = phase; ϕ is called initial phase.

Characteristics of S H M

A) Instantaneous displacement: The distance of the particle from mean position in a particular direction at any instant of time is known as instantaneous displacement.

It is given by $Y = A \sin(\omega t + \phi)$

If the particle starts from Mean position, $\phi = 0$ then

i) $Y = 0$, at Mean position

ii) $Y = A$ at extreme position

B) Velocity: The rate of change of displacement is called velocity.

$$v = \frac{dy}{dt} = Aw\cos(wt + \phi) \quad \text{and} \quad v = \omega\sqrt{A^2 - y^2}$$

If the particle starts from the mean position, $\phi = 0$ then

i) $v = A\omega$, ie., maximum at Mean Position

ii) $v = 0$, ie., minimum at extreme Position

C) Acceleration: the rate of change of velocity of a particle in S H M is called acceleration.

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 y \quad \text{or} \quad a \,\alpha\, -y$$

If the particle starts from the Mean position, $\phi = 0$, then

i) $a = 0$, i.e., minimum at mean position

ii) $a = \omega^2 A$, i.e., maximum at extreme position.

D) Time Period: Time taken by vibrating particle in S.H.M. to complete one vibration is called Time period or period of oscillation.

General formula: $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{y}{a}}$

E) Potential Energy

$$P.E. = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2\theta$$

Where m = mass of S.H.M.

x = displacement of S.H.M. from its mean position

A = amplitude of oscillation

θ = phase angle from its mean position

During one complete vibration average potential Energy is given by = $1/4\ m\omega^2 A^2$

F) Kinetic Energy

The K. E. of a particle in S.H.M is given by K. E = $1/2\ m\omega^2 (A^2 - y^2) = 1/2\ m\omega^2 A^2 \cos^2(\omega t \pm \phi)$

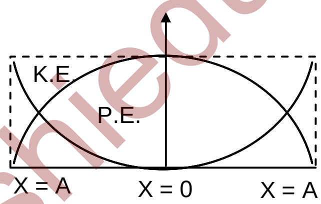
During one complete vibration average kinetic Energy = $1/4\ m\omega^2 A^2$

G)Total Energy

i) T. E. = P. E. + K. E. = $1/2\ m\omega^2 A^2 + U_0$

ii) When a particle is in S. H. M. At any position T. total energy is constant.

Energy and displacement curve.



11. Equilibrium Position: the point at which no net force acts on the oscillating body is known as equilibrium position or mean position.

i) Displacement of the body is Minimum.

ii) Velocity of the body is Maximum.

iii) Acceleration of the body is Minimum.

iv) P. E. of the body is Minimum.

v) At the Mean position K.E. of the body is Maximum

12. Extreme Position: the point at which maximum force acts on the oscillating body is known as extreme Position.

i) Displacement of the body is Maximum.

ii) Velocity of the body is Minimum.

iii) Acceleration of the body is Maximum.

iv) P. E. of the body is Maximum.

v) K. E. of the body is Minimum.

13. If V_1 and V_2 are the velocities at displacements y_1 and y_2, then $A = \frac{V_1^2\, y_2^2 - V_2^2\, y_1^2}{V_1^2 - V_2^2}$

$\omega = \left(\frac{V_1^2 - V_2^2}{y_2^2 - y_1^2}\right)$ and $T = 2\pi\sqrt{\frac{y_2^2 - y_1^2}{V_1^2 - V_2^2}}$.

14. If f is the frequency of SHM, then the frequency of kinetic energy or potential energy is 2f.