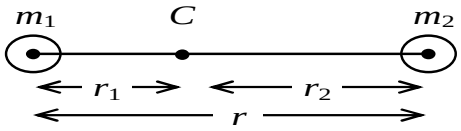
Centre of Mass

1. Centre of mass is the imaginary point at which the total mass of the system is supposed to be concentrated.
2. There need not be any mass at the centre of mass. Ex.: Hollow sphere, ring etc
3. Internal forces, however strong they are, cannot produce acceleration in centre of mass.
4. When a force is applied along a line passing through the centre of mass, the entire system moves in linear motion.
5. If no external force acts on a system, the acceleration of centre of mass is zero, the velocity and momentum of the centre of mass remains constant, though velocity and momentum of individual particles vary.
6. The motion of the centre of mass can be studied using Newton's laws of motion.
7. The algebraic sum of moments of masses of all the particles about the centre of mass is always zero.
8. Couple cannot change the position of centre of mass.
9. An imaginary point at which the total weight of the system is supposed to be concentrated is known as centre of gravity.
10. For small objects c.m. and c.g. coincide but for large or extended objects like hills, buildings they do not coincide.
11. At the centre of the earth there is no centre of gravity $(\because g = 0)$
12. The position of center of mass depends on the shape of the body and distribution of mass.
13. The location of the centre of mass is independent of the reference frame used to locate it.
14. The centre of mass of a system of particles depends only on the masses of the particles and their relative positions.
15. Centre of mass of a system is unaffected, with the change of the co-ordinate system.
16. If the system consists of n particles CM is given by $x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n}$
17. If the system of particles lies along y-axis then $y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots + m_n y_n}{m_1 + m_2 + \ldots + m_n}$
18. If the system of particles lies along z-axis then $z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \ldots + m_n z_n}{m_1 + m_2 + \ldots + m_n}$
19. Velocity of centre of mass is $V_{cm} = \frac{m_1 V_1 + m_2 V_2 + \ldots}{m_1 + m_2 + \ldots}$

20. Acceleration of CM $a_{cm} = \frac{m_1a_1 + m_2a_2 + ...}{m_1 + m_2 + ...}$

21. Two particles of masses m_1, m_2 are separated by a distance r, then distance of centre of mass is



$r_1 = \frac{m_2 r}{m_1 + m_2}$ and $r_2 = \frac{m_1 r}{m_1 + m_2}$

The centre of mass will be nearer to the particle of more mass. If $m_1 = m_2$, then $r_1 = r_2$.

22. Two spheres of same material r_1 and r_2 are kept in contact, distance of C.M from the centre of Ist sphere is equal to $d_1 = \left(\frac{r_2^3}{r_1^3 + r_2^3}\right)(r_1 + r_2)$ and $d_2 = \frac{r_1^3}{r_1^3 + r_2^3}(r_1 + r_2)$

[d_2 – distance of C.M from centre of second sphere]

23. In the above case, instead of spheres they are circular discs then

$d_1 = \left(\frac{r_2^2}{r_1^2 + r_2^2}\right)(r_1 + r_2)$ and $d_2 = \frac{r_1^2(r_1 + r_2)}{(r_1^2 + r_2^2)}$

24. When a portion of m_2 is removed from a body of mass m_1 then shift in the position of center of mass $(x) = \frac{\left[\begin{matrix}distance\ between\ cm\ of \\ the\ body\ and\ removed\ part\end{matrix}\right] \times mass\ of\ removed\ part}{mass\ of\ remaining\ part}$

25. From a disc of radius R, a disc of radius r is removed from one end then shift in centre of mass is $x = \frac{r^2(R-r)}{(R^2 - r^2)}$.

26. From a uniform solid sphere of radius R, a sphere of radius r is removed from one end, and then the shift in CM is $x = \frac{r^3(R-r)}{(R^3 - r^3)}$.

27. When a person walks on a boat in still water, centre of mass of person, boat system is not displaced.

a) If the man walks a distance L on the boat, the boat is displaced in the opposite direction relative to shore or water by a distance $x = \frac{mL}{M+m}$.

b) Distance walked by the mass relative to shore or water is (L - x).