Electric Potential

Synopsis

1. **Electric potential (V)**

   i) Electric potential at a point in a field is the amount of work done in bringing a unit +ve charge from infinity to the point.

   ii) It is equal to the Electric potential energy of unit + ve charge at that point.

   iii) It is a scalar.

   iv) S.I unit is volt.

   v) Potential at a distance'd' due to a point charge q in air or vacuum is $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d}$

   vi) $V = -\int \vec{E}.\vec{d}x$.

   vii) $\vec{E} = -\frac{dv}{dx}$ (or) $V = Ed$.

   viii) A positive charge in a field moves from high potential to low potential where as electron moves from low potential to high potential when left free.

   ix) Work done in moving a charge q through a potential difference V is W = q V joule

   x) Gain in the Kinetic energy; $\frac{1}{2}mv^2 = qV$.

   xi) Gain in the velocity $v = \sqrt{\frac{2qV}{m}}$.

2. **Equipotential surface**

   i) A surface on which all points are at the same potential

   ii) Electric field is perpendicular to equipotential surface

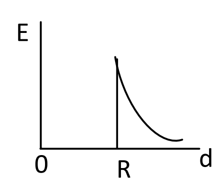
   iii) Work done in moving a charge on equipotential surface is zero.

3. **In the case of a hollow charged sphere.**

   i) Intensity at any point inside the sphere is zero.

   ii) Intensity at any point on the surface is same and it is maximum

   $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$



iii) Outside the sphere $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d^2}$ d = distance from the centre

iv)It behaves as if the whole charge is at its centre.

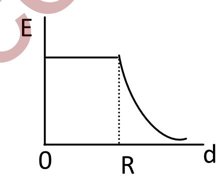
v) Electric field Intensity in vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d^3}\vec{d}$ or $\vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d^2}\hat{d}$

vi)The resultant electric field intensity obeys the principle of superposition.

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

4. **In the case of hollow charged sphere**

i) The potential at any point inside the sphere is same as that at any point on its surface



$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

ii) It is an equipotential surface.

iii) Outside the sphere, the potential varies inversely as the distance of the point from the centre.

$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d}$

Note: Inside a non conducting charged sphere electric field is present.

Electric intensity inside the sphere

$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}d$

Here d is the distance from the centre of sphere.

$E \propto d$

5. **Electron volt**

i) This is the unit of energy in particle physics and is represented as eV.

ii) 1 eV = 1.602×10^{-19} J.

6. **Charged particle in electric field**

1) When a positive test charge is fired in the direction of an electric field,

i) It accelerates,

ii) Its kinetic energy increases and hence

iii) Its potential energy decreases.

2) A charged particle of mass m carrying a charge q and falling through a potential V acquires a speed of $\sqrt{2Vq/m}$.

7. **Electric dipole**

i) Two equal and opposite charges separated by a constant distance is called electric dipole. $\vec{P} = q.2\vec{l}$.

ii) **Dipole moment ($\vec{P}$)** is the product of one of the charges and distance between the charges. It is a vector directed from negative charge towards the positive charge along the line joining the two charges.

iii) The torque acting on an electric dipole placed in a uniform electric field is given by the relation $\vec{\tau} = \vec{P} \times \vec{E}$ i.e., $\tau = PE\sin\theta$, where θ is the angle between $\vec{P}$ and $\vec{E}$.

iv) The electric intensity (E) on the axial line at a distance'd' from the centre of an electric dipole is $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Pd}{(d^2 - l^2)^2}$ and on equatorial line, the electric intensity (E) = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{(d^2 + l^2)^{3/2}}$.

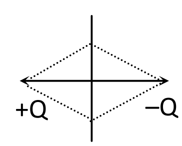
v) For a short dipole i.e., if $l^2 << d^2$, then the electric intensity on axial line is given by $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2P}{d^3}$.

vi) For a short dipole i.e., if $l^2 << d^2$, then the electric intensity on equatorial line is given by

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{d^3}$.

vii) The potential due to an electric dipole on the axial line is

$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{(d^2 - l^2)}$ and at any point on the equatorial line it is zero.



viii) Two unlike equal charges +Q and –Q are separated by distance

1) The net electric potential is zero on the perpendicular bisector of the line joining the charges.

2) The bisector is equipoptential and zero potential line.

3) Work done in moving a charge on this line is zero.

4) Electric intensity at any point on the bisector is perpendicular to the bisector.

5) Electric intensity at any point on the bisector parallel to the bisector is zero.

8. **Electric potential energy**

i) A charge placed in an electric field possesses potential energy and is measured by the work done in moving the charge from infinity to that point against the electric field.

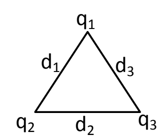
ii) If two charges q_1 and q_2 are separated by a distance d, the P.E. of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}$$

iii) If two like charges (two protons or two electrons) are brought towards each other, the P.E of the system increases.

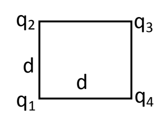
iv) If two unlike charges (a proton and an electron) are brought towards each other, the P.E. of the system decreases.

v) If three charges q_1, q_2 and q_3 are situated at the vertices of a triangle (as shown in the figure), the P.E. of the system is



$$U = U_{12} + U_{23} + U_{31} \quad = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d_1} + \frac{q_2 q_3}{d_2} + \frac{q_3 q_1}{d_3}\right)$$

vi) If four charges q_1, q_2 , q_3 and q_4 are situated at the corners of a square as shown in the figure, P.E of the system



$$\frac{1}{4\pi\varepsilon_0}\times\left(\frac{q_1 q_2}{d} + \frac{q_2 q_3}{d} + \frac{q_3 q_4}{d} + \frac{q_4 d_1}{d} + \frac{q_2 q_4}{\sqrt{2}d} + \frac{q_1 q_3}{\sqrt{2}d}\right)$$

vii) In the field of a charge Q, if a charge q is moved against the electric field from a distance 'a' to a distance 'b' from Q, the work done W is given by

$$W = (V_b - V_a)q = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{b} - \frac{1}{4\pi\varepsilon_o}\frac{Qq}{a} = \frac{Qq}{4\pi\varepsilon_o}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{Qq}{4\pi\varepsilon_o}\left[\frac{a-b}{ab}\right]$$

9. **Combined field due to two point charges**

a) Due to two similar charges

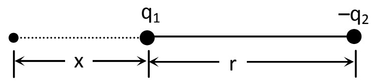
i) If charges q_1 and q_2 are separated by a distance 'r', null point (where resulting field intensity is zero) is formed on the line joining those two charges.

ii) Null point is formed within the charges.

iii) Null point is located nearer to weak charge.

iv) If x is distance of null point from q_1,

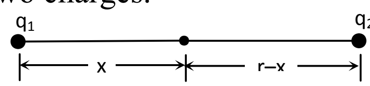
(Weak charge) then $\frac{q_1}{x^2} = \frac{q_2}{(r-x)^2}$



$\Rightarrow x = \frac{r}{\sqrt{q_2/q_1}+1}$ Here q_1 and q_2 are like charges

b) Due to two dissimilar charges

i) If q_1 and q_2 are unlike charges then null point is formed on the line joining two charges.



ii) Null point is formed outside the charges.

iii) Null point is form nearer weak charge.

iv) x is the distance of null point from q_1(weak charge) then $\frac{q_1}{x^2} = \frac{q_2}{(r+x)^2}$

$$\Rightarrow x = \frac{r}{\sqrt{q_2/q_1}-1}$$

In the above formulae q_2/q_1 is numerical ratio of charges

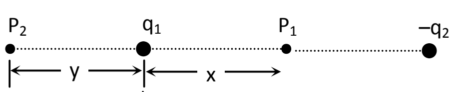
c) Zero potential point due to two charges

i) If two unlike charges q_1 and q_2 are separated by a distance 'r', the net potential is zero at two points on the line joining them.

ii) One in between them and the other outside the charges.

iii) Both the points are nearer to weak charge (q_1).

$\frac{q_1}{x} = \frac{q_2}{(r-x)}$ (For point 1, within the charges)



$\frac{q_1}{y} = \frac{q_2}{(r+y)}$ (For point 2, outside the charges)

Here q_2 is numerical value of strong charge

$$\Rightarrow x = \frac{r}{\frac{q_2}{q_1}+1} \; ; \; y = \frac{r}{\frac{q_2}{q_1}-1}$$

d) Due to two similar charges zero potential point is not formed.

10. **Equipotential surface**

i) The surface which is the locus of all points which are at the same potential is known as equipotential surface

ii) No work is required to move a charge from one point to another on the equipotential surface.

iii) No two equipotential surfaces intersect

iv) The direction of electric lines of force or direction of electric field is always normal to the equipotential surface.

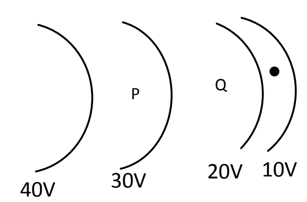
v) Inside a hollow charged spherical conductor the potential is constant. This can be treated as equipotential volume. No work is required to move a charge from the centre to the surface.

vi) For an isolated point charge, the equipotental surface is a sphere. I.e. concentric spheres around the point charge are different equipotential surfaces.

vii) In a uniform electric field any plane normal to the field direction is an equipotential surface.

viii) The spacing between equipotential surfaces enables us to identify regions of strong and weak field.

$$E = -\frac{dV}{dr} \Rightarrow E \propto \frac{1}{dr}$$



$E_P < E_Q < E_R$