ENGINEERING MECHANICS

LAWS OF FORCES

Polygon Law of Forces:

Definition: If number of forces acting at a point is represented in magnitude and direction by the sides of polygon, taken in order in the closing side represents the resultant in magnitude and direction taken in opposite order.

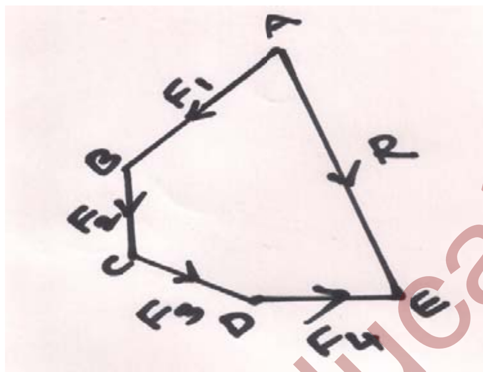

Figure: 1 Polygon Law of Forces

- From above figure, F1, F2, F3 and F4 are the side forces of the polygon and the force R is resultant of all the forces

Resolution of Force:

Definition: The force which is replaced by two components which are equivalent to the given force is called resolution of forces.

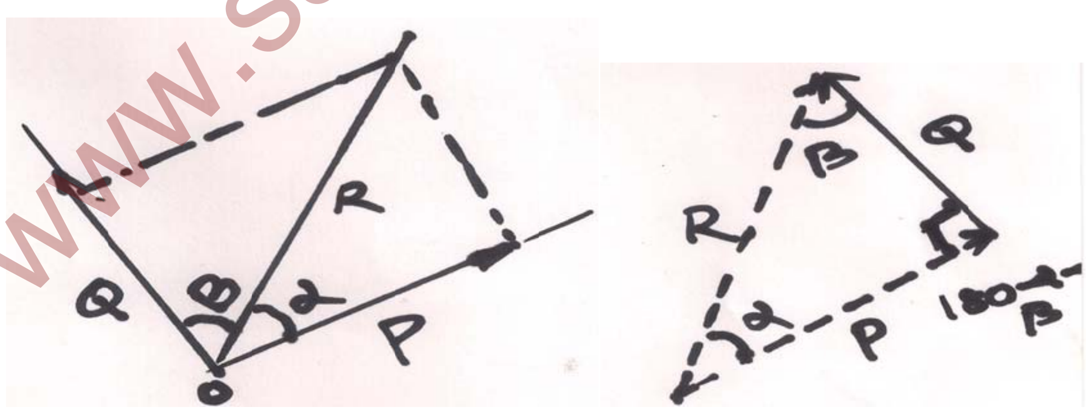

Figure 2 Resolution of forces

Forces P and Q are the components of the resultant force R, the above given figure is oblique components of resultant force.

This kind of problems can be solved easily by sine rule of mathematics as follows-

$$\frac{R}{\sin(180-\alpha-\beta)} = \frac{P}{sin\beta} = \frac{Q}{sin\alpha}$$

Rectangular Components:

From figure resolution of forces along the X and Y axis are F_x and F_y respectively. These forces F_x and F_y are called rectangular component forces of resultant force F as shown in below figure.

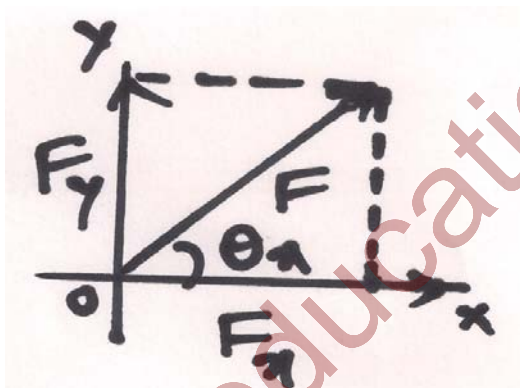

Figure 3 Rectangular components of forces

From above figure:

Where F=Resultant force

Fx= X- component of t he force F

Fy= Y-component of the force F

Angle Ø is the angle of force F with X- axis

$$\mathbf{Cos\emptyset = \frac{Fx}{F} \quad ; \quad Fx = F\,.\,Cos\emptyset}$$

$$\mathbf{Sin\emptyset = \frac{Fx}{F} \quad ; \quad Fx = F\,.\,Sin\emptyset}$$

IF angle β is the angle of force F with Y- axis

$$\mathbf{Sin\beta = \frac{Fx}{F} \quad ; \quad Fx = F\,.\,Sin\beta}$$

$$\mathbf{Cos\beta = \frac{Fy}{F} \quad ; \quad Fy = F\,.\,Cos\beta}$$

Resultant force F can be calculated by using Pythagoras theorem:

$$\boldsymbol{F = \sqrt{(}Fx^2 + Fy^2}$$

Let us solve some problems pertaining to coplanar forces:

Q. 1. Determine the X and Y components of the each force

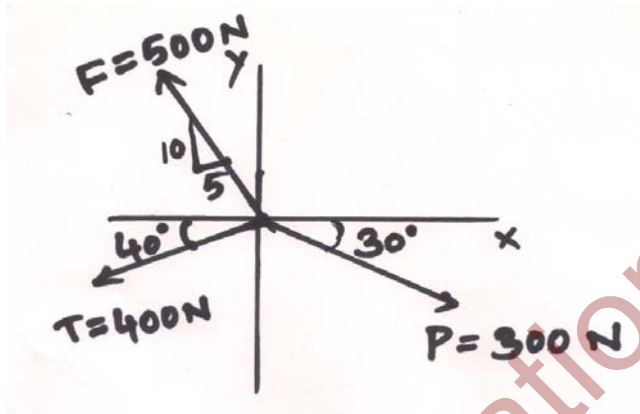

Figure 4: Coplanar forces diagram

1. Calculation for components of force F in X and Y directions

$$\frac{\boldsymbol{Fx}}{\mathbf{-5}} = \frac{\boldsymbol{Fy}}{\mathbf{10}} = \frac{\mathbf{500}}{\sqrt{\mathbf{5^2 + 10^2}}}$$

$$\boldsymbol{Fx} = \mathbf{-223.606\ N}$$

$$\boldsymbol{Fy} = \mathbf{447.21}\ \boldsymbol{N}$$

2. Calculation for components of force T in X and Y directions

Tx = -T Cos 40

= -400 Cos 40 = **-306.42N**

Ty= -T Sin 40

= -400 Sin 40 = **-257.11 N**

3. Calculation for components of force P in X and Y directions

Px= P Cos 30

=300 Cos 30=**259.81 N**

Py = P Sin 30
= 300 Sin 30 = **-150 N**

Q. 2. Compute the X and Y components of each force (P, F, Q, T) given in the figure.

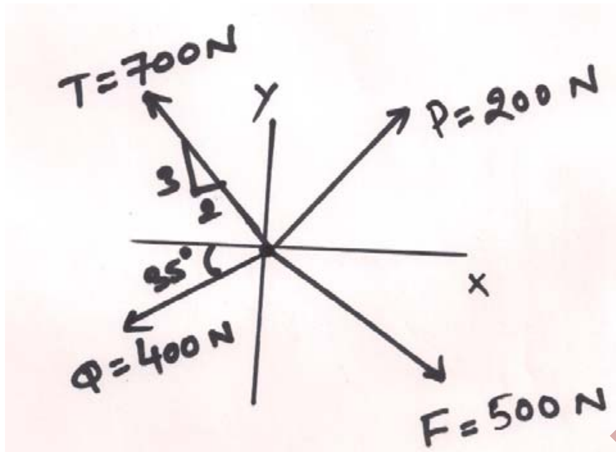

Figure 5: Coplanar forces diagram

1. Let us find the X and Y components of force P

   Px = 200 Cos 60 = 100 N

   Py = 200 Sin 60 = 173.2

2. Let us find the X and Y components of force T

$$\frac{\boldsymbol{Tx}}{\mathbf{-2}} = \frac{\boldsymbol{Ty}}{\mathbf{3}} = \frac{\mathbf{700}}{\sqrt{\mathbf{2^2+3^2}}}$$

Tx=-388.29 N

Ty=582.43 N

3. Let us find the X and Y components of force Q

   Qx=-400 Cos 35=**-327.66**

   Qy=-400 Sin 35 =**-229.43**

4. Let us find the X and Y components of force F

$$\frac{\boldsymbol{Fx}}{\mathbf{2}} = \frac{\boldsymbol{Fy}}{\mathbf{-1}} = \frac{\mathbf{500}}{\sqrt{\mathbf{2^2+1^2}}}$$

Fx=447.21 N

Fy=223.60 N

Q. 3. The body on the inclined surface as shown in figure is subjected to the vertical and horizontal forces (mention in the diagram). Find the components of force along the X and Y axis oriented parallel and perpendicular to the incline.

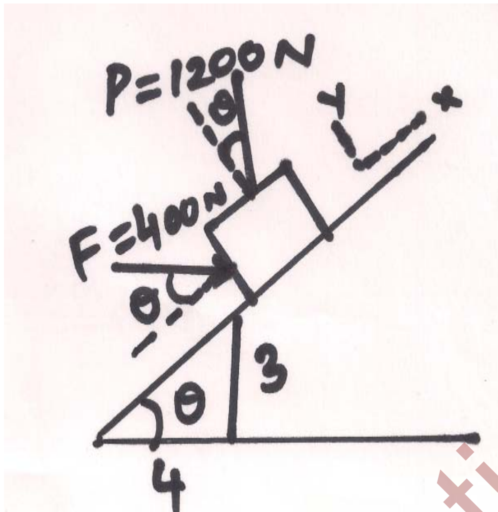

Figure 6: The body in the inclined surface

Angle of the slope can be calculate as below

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$

<u>Angle of force F with X -axis is θ</u>

Force F in X- direction, Fx = F Cos θ

=400 Cos 36.87 =**320 N**

Force F in Y- direction, Fy = F Sin θ

=400 Sin 36.87 =240 N

<u>Angle of force P with Y -axis is θ</u>

Force P in X- direction, Px = P Sin θ

=-1200 Sin 36.87 =-720 N

Force P in Y- direction, Py = P Cos θ

=-1200 Cos 36.87 =-960 N

<u>Components of forces in space:</u>

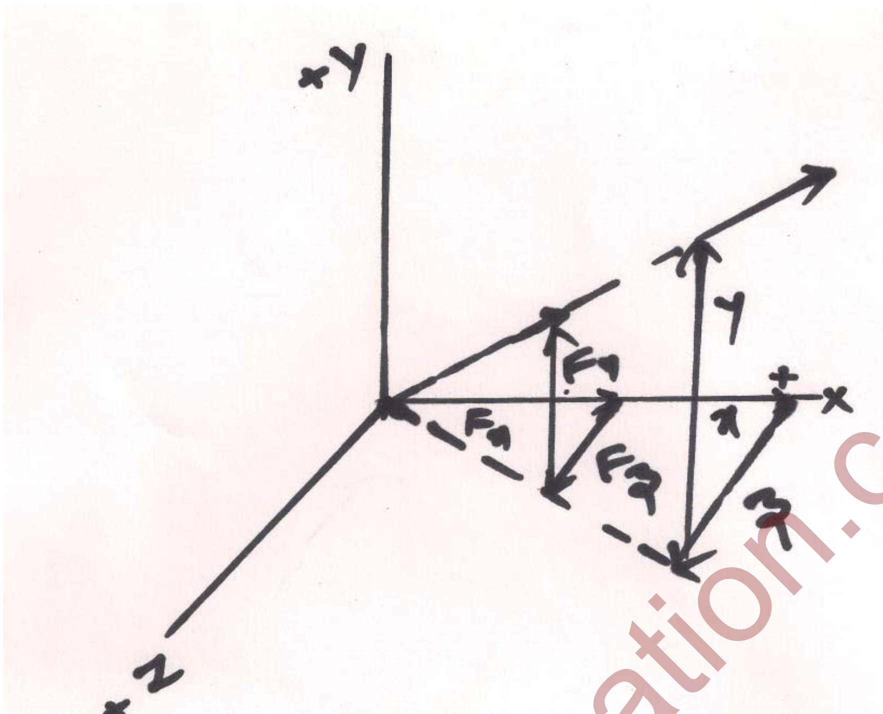

Figure 7: Representation of Components of force in space

To determine the rectangular components of a space force consider showing a force F whose direction specified by two points along its line of action. The rectangular components of force are directly proportional to the rectangular components of the distance d separating the two points.

This proportionality is an extension of the relations between the sides of a fore triangle.

Force Multiplier: It is the value of the equal ratios and is known as the multiplier if the force F (or more briefly, as force multiplier) expressed in units of fore per unit length.

$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = F_m$$

it is easy to show by repeated applications of the Pythagorean theorem that the magnitudes of F and d are given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \text{ and } \quad d = \sqrt{(x^2 + y^2 + z^2}$$

If at all in the problem θ_x , θ_y *and* θ_z are given as the angles of force F with X, Y and Z axis respectively then the components of force F can be calculated as follows

$$F_x = F \, Cos \, \theta_x \quad : \quad \frac{F_x}{Cos \, \theta_x} = F$$

$$F_y = F \, Cos \, \theta_y \quad : \quad \frac{F_y}{Cos \, \theta_y} = F$$

$$F_z = F \, Cos \, \theta_z \quad : \quad \frac{F_z}{Cos \, \theta_z} = F$$

Above equations can be rearranged as below:

$$\frac{F_x}{Cos \, \theta_x} = \frac{F_y}{Cos \, \theta_y} = \frac{F_z}{Cos \, \theta_z} = F$$

It should be noted that the direction angles are not independent. Fact that the sum of the squares of the components of a force is equal to square of its magnitude, the above equation can be reduced to

$$Cos^2 \, \theta_x + Cos^2 \, \theta_y + Cos^2 \, \theta_z = 1$$

Let us solve a problem on components of forces in a space:

Q. 4 Find the resultant of the concurrent force system shown in fig. which consists of the forces T=300 N, P=200 N and F=500 N directed from D towards A, B and C respectively.

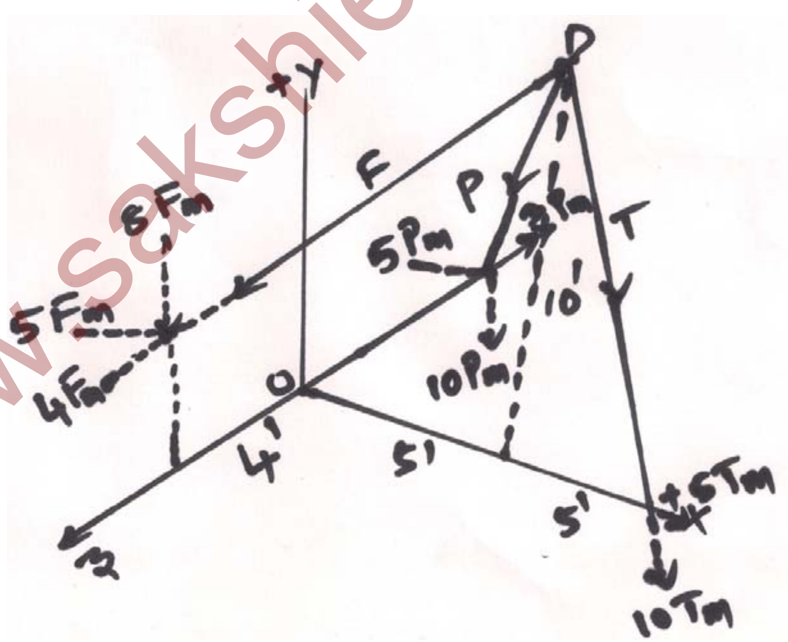

Figure 8: Components of force in special coordinates

Sol. To solve this kind of problems, better to tabulate all the forces and their distances and this table is almost self explanatory.

The components of distances are the signed lengths travelled from D along coordinate paths to A, B and C.

- The term in the column headed Distance d are found from the equation

$$d = \sqrt{(x^2 + y^2 + z^2}$$

- The force multiplier is the ratio of force to distance

$$\frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z} = F_m$$

- Corresponding force components can be calculated by multiplying this force multiplier with the corresponding distance.

$$F_x = x.\ F_m \quad , \quad F_y = y.\ F_m \quad \text{and} \quad F_z = z.\ F_m$$

all above parameters are tabulated below for simplify the calculations to find the force components

Force (N)	Comps. of distance (m)			Distance (m)	Force multiplier (N/m)	Force components (N)		
	x	y	z			X Comp.	Y Comp.	Z Comp.
T=300	5	-10	0	11.18	26.8	134	-268	0
P=200	-5	-10	-3	11.57	17.3	-86.40	-173	-51.9
F=500	-5	-8	4	10.25	48.8	-244	-390	195
Totals						-196.4	-831	143.1

From the algebraic summation of force components obtained

Resultant force in X − diretion: $R_x = \sum X = -196.4$ N

Resultant force in Y − diretion: $R_y = \sum Y = -831$ N

Resultant force in Z − diretion: $R_z = \sum Z = 143.1$ N

Cumulative Resultant force of the system $\boldsymbol{R} = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$$\boldsymbol{R} = \sqrt{196.4^2 + 831^2 + 143.1^2}$$

R=865 N

Therefore magnitude of the resultant force R= 865 N

This resultant force direction cosines and direction angles are determined as follows

$$\frac{R_x}{Cos\ \theta_x} = \frac{R_y}{Cos\ \theta_y} = \frac{R_z}{Cos\ \theta_z} = R$$

$$\frac{196.4}{Cos\ \theta_x} = \frac{831}{Cos\ \theta_y} = \frac{143.1}{Cos\ \theta_z} = 865$$

$$\frac{196.4}{Cos\ \theta_x} = 865$$

$$Cos\ \theta_x = \frac{196.4}{865}$$

$$\theta_x = \cos^{-1}\frac{196.4}{865}$$

$$\theta_x = 76.88°$$

Similarly we can determine the angles of θ_y *and* θ_z

Therefore θ_y **=16.1°**

θ_z**=80.48°**

Note:-*Now we will try to solve the Exercise problem in simplified manner from F L Singer (problem No:2-5.6, Page No:33). While solving this problem we are going to take pounds (lb) as Newton (N) and inches (in) as meters (m).*

Q. 5. Determine the resultant of the system of concurrent forces having the following magnitudes and passing through the origin and the indicated points: P=280 N (+12, +6, -4); T=520 N (-3, -4, +12); F=270 N (+6, -3, -6).

Sol. To solve this problem we will generate a table, to solve simplified manner and using above formulae.

Force (N)	**Comps. of distance (m)**			**Distance (m)**	**Force multiplier (N/m)**	**Force components (N)**		
	x	**y**	**z**			**X Comp.**	**Y Comp.**	**Z Comp.**
P=280	+12	+6	-4	14	20	240	120	-80
P=520	-3	-4	+12	13	40	-120	-160	480
F=270	+6	-3	-6	9	30	180	-90	-180
Totals						300	-130	220

Resultant force in X − diretion: $R_x = \sum X = 300$ N

Resultant force in Y − diretion: $R_y = \sum Y = -130$ N

Resultant force in Z − diretion: $R_z = \sum Z = 220$ N

Cumulative Resultant force of the system $\boldsymbol{R} = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$$\boldsymbol{R} = \sqrt{300^2 + 130^2 + 220^2}$$

Therefore Magnitude of Resultant force: R=394.08 N

Direction of Resultant force can be decided based on the signs of R_x R_y *and* R_z

R_x value is positive means Resultant force (R) is pointing ***forward***

R_y value is negative means Resultant force (R) is pointing ***downward***

R_z value is positive means Resultant force (R) is pointing ***right***

Sample calculation for reference: for force 'P'

x, y and z are the co-ordinates of force P acting from the origin.

Given P=280 N (+12, +6, -4) means

Magnitude of force P=280 N

x=12, y=6 and z=-4

$$\boldsymbol{d} = \sqrt{(\boldsymbol{x^2} + \boldsymbol{y^2} + \boldsymbol{z^2}} = \sqrt{(\mathbf{12^2} + \mathbf{6^2} + \mathbf{4^2}}$$

d=14 m

Force Multiplier $\boldsymbol{P_m} = \frac{\boldsymbol{P}}{\boldsymbol{d}} = \frac{\mathbf{280}}{\mathbf{14}} =$ **20 N/m**

X Component of force P=$P_m.x$=20x12=240 N

Y Component of force P=$P_m.y$=20x6=120 N

Z Component of force P=$P_m.z$=20x-4=-80 N