# **Moment of Inertia**

Moment of inertia of a body is independent of

1) Mass of the body

2) Distribution of mass of the body

3) Temperature of the body

4) Angular velocity of the body

Moment of inertia of a body depends upon

1) Distribution of mass of the body

2) Position of axis of rotation

3) Temperature of the body

4) All of the above

3. If I<sub>1</sub> I<sub>2</sub>, and I<sub>3</sub> are moments of inertia of a disc about its geometric axis, diameter and a tangent in its plane, then

1)  $I_1 > I_2 > I_3$ 

2)  $I_3 > I_2 > I_1$ 

3)  $I_3 > I_1 > I_2$  4)  $I_2 > I_1 > I_3$ 

Moment of inertia of a ring is minimum

1) About its geometric axis

2) About a diameter

3) About a tangent in its plane

4) Tangent perpendicular to its plane

Moment of inertia of a solid cylinder of mass m, length l, and radius r about an axis passing through its centre and perpendicular to its length is

2)  $\frac{ml^2}{3}$ 

3)  $\frac{ml^2}{12} + \frac{mr^2}{4}$  4)  $\frac{ml^2}{12} + \frac{mr^2}{3}$ 

6. Moment of inertia of a uniform plate of mass M, length 'l' and breadth 'b' about an axis passing through its centre of mass and normal to the plane is

1) M  $(l^2+b^2)/2$  2) M  $(l^2+b^2)/12$  3) M  $(l^2+b^2)/3$  4) M $(l^2+b^2)$ 

7. The moment of inertia of a thin uniform rod of mass M and length l about an axis perpendicular to the rod, through its centre is l. The moment of inertia of the rod about an axis perpendicular to the rod through its end point is

1) 1/4

2) 1/2

3) 21

4) 41

8. The correct relation of the following is

1)  $\bar{\tau} = \bar{r} \cdot \bar{F}$ 

2)  $\bar{\tau} = \bar{r} \times \bar{F}$ 

3)  $\bar{\tau} = \frac{\overline{F}}{\overline{r}}$ 

4)  $\vec{\tau} = \vec{r} + \vec{F}$ 

9. When a constant torque is applied on a rigid body, then

1) The body moves with linear acceleration

- 2) The body rotates with constant angular velocity
- 3) The body rotates with constant angular acceleration
- 4) The body undergoes equal angular displacement in equal intervals of time
- 10. When a rigid body is rotating, every point in it describes a circular path
  - 1) Only if the axis of rotation passes symmetrically through the centre of gravity of the body
  - 2) Only it the axis of rotation passes symmetrically through the body
  - 3) Provided the axis passes through the edge of the body
  - 4) Whatever be the axis of oration
- 11. A solid metal sphere and a solid wooden sphere are having the same mass. If they are spinning with same angular velocity, then
  - 1) Metal sphere possesses more angular momentum
  - 2) Wooden sphere will have more angular momentum
    - 3) Both will have same angular momentum
    - 4) None of the above

12.	If polar ice caps melt, the length of the day				
	1) Will increase 2) Will o	decrease	3) Will remain same	4) Cannot be decided	
13.	A boy standing on a rotating table with heavy spheres in his hands suddenly				
	brings his hands close to his body. The angular velocity of the table				
	1) Remains unchanged		2) Becomes Z	ero	
	3) Decreases		4) Increases		
14.	A solid sphere, solid cylind	er and a d	lisc are allowed to rol	l down from the top of	
	an incline plane from the same height. Then				
	1) Disc will reach the botton	n first			
	2) Solid cylinder will reach t	he bottom	first	<b>)</b>	
	3) Sphere will reach the bott	om first			
	4) All will reach the bottom	simultaneo	ously		
15.	Two identical hollow spheres roll down two inclined planes of same height but				
	of different angles of inclination. Then, they reach the bottom				
	1) With same speeds and in	same time			
	2) With different speeds and in different times				
	3) With same speed but in different times				
	4) With different speeds in same time				
16.	Moment of inertia of a rigid body depends on				
	A) Mass of body				
	B) Position of axis of rotation				
	C) Angular velocity of the body				
	D) Time period of its rotation				
	1) A and B are correct		2) B and C are	e correct	
	3) A and C are correct		4) C and D are	e correct	

<b>17.</b>	Identify the correct order in which the values of M.I. decreases for the following			
	i. M.I. of solid sphere of mass 'M' and radius 'R' about its diameter of rotation			
	ii. M.I. of uniform	ring of mass 'M'	and radius 'R'	about its tangent
	perpendicular to its p	lane		
	iii. M.I. of uniform di	sk of mass 'M' and r	adius 'R' about its	diameter
	iv. M.I of a uniform s	olid cylinder of mass	M about its own a	axis of rotation
	1) iii, i, iv, ii	2) i, iv, iii, ii	3) i, ii, iii, iv	4) iv, iii, ii, i
18.	Four objects with the	he same mass and	radius are spini	ning freely about a
	diameter with the san	ne angular speed. Aı	range the work r	equired to stop them
	in the decreasing orde	er		
	a) Solid sphere	b) Hollow sphere	c) Disc	d) Hoop
	1) d, b, c, a	2) a, b, c, d	3) b, a, d, c	4) c, a, b, d
19.	. A thin disc rotates about an axis passing through its centre and perpendicular to			and perpendicular to
	its plane with a cons	tant angular velocit	" I is the mome	ent of inertia of that
	disc and 'L' is its angular momentum about the given axis. Then rotational			
	kinetic energy of the disc 'E' is			
	A) E $\alpha$ L <sup>2</sup>	15,	B) E $\alpha$ L <sup>-2</sup>	
	C) Ε α Ι		D) $\mathbf{E} \alpha \mathbf{I}^{-1}$ (if L constant)	
	1) A and C are correct			
	2) B and C are correct			
	3) B and D are correct			
	4) A and D are correct			

- 20. A):  $I_S$  and  $I_H$  are moment of inertia about the diameters of a solid and thin-walled hallow sphere respectively. If the radius and masses of above spheres are equal,  $I_H > I_S$ .
  - R): In solid sphere, the mass is continuously and regularly distributed about the centre, where as the mass, to a large extent, is concentrated on the surface of hollow sphere.
  - 1) Both (A) and (R) are true and (R) is the correct explanation of (A).
  - 2) Both (A) & (R) are true but (R) is not correct explanation of (A).
  - 3) (A) is true and (R) is false.
  - 4) (A) is false but (R) is true.
- 21. (A): The angular momentum of a particle with respect to origin moving parallel to x-axis with constant velocity is constant.
  - (R): There is no change in the perpendicular distance of the particle from the origin.
  - (1) Both (A) and (R) are true and (R) is the correct explanation of (A).
  - (2) Both (A) and (R) are true and (R) is not the correct explanation of (A).
  - (3) (A) is true but (R) is false.
  - (4) (A) is false but (R) is true.
- 22. (A): A gymnast curls his body while diving to perform more number of somersaults in air.
  - (R): Curling reduces his moment of inertia and increases angular velocity.
  - (1) Both (A) and (R) are true and (R) is the correct explanation of (A).
  - (2) Both (A) and (R) are true and (R) is not the correct explanation of (A).
  - (3) (A) is true but (R) is false.
  - (4) (A) is false but (R) is true.

- 23. (A): If a raw egg and a boiled egg are spinning on the table by applying same torque then raw egg comes, to rest quickly.
  - (R): While spinning, moment of inertia of boiled egg will be less than that of raw egg.
  - (1) Both (A) and (R) are true and (R) is the correct explanation of (A).
  - (2) Both (A) and (R) are true and (R) is not the correct explanation of (A).
  - (3) (A) is true but (R) is false.
  - (4) (A) is false but (R) is true.
- 24. Match list I with list II

- a) Torque
- b) Angular momentum
- c) Rotational K.E

- List II
- e) (1/2) L
- f) Zero
- $\mathbf{g}$ )  $I\alpha$
- d) Work done by centripetal force h) Iw
- (1) a-h, b-e, c-f, d-g
- (2) a-f, b-h, c-g, d-e
- (3) a-e, b-f, c-g, d-h
- (4) a-g, b-h, c-e, d-f
- 25. If M is the mass and R the radius, match the Moment of Inertia for the given objects about the indicated axes

**Object** 

### **Moment of Inertia**

(a) Circular Ring

(e)  $\frac{7}{5}MR^2$  about tangent

(b) Hollow Sphere

(f)  $\frac{5}{4}MR^2$  about tangent  $\perp$  to plane

(c) Circular Disc

(g) 2MR<sup>2</sup> about tangent, to plane

(d)Solid Sphere

(h)  $\frac{5}{3}MR^2$  about tangent

(2) a-g, b-h, c-f, d-e

(4) a-f, b-h, c-g, d-e

26. The radius of gyration of a body about an axis at a distance of 4 cm from its

2) 5 cm

centre of mass is 5 cm. The radius of gyration about a parallel axis through

3) 4 cm

(1) a-g, b-e, c-g, d-f

(3) a-f, b-g, c-e, d-h

centre of mass is

1) 2 cm

27.	The moment of inertia	a of a cylinder about	its own axis is equal	to its moment of	
	inertia about an axis passing through its centre and normal to its length. The				
	ratio of length to radio	us is			
	1) 2: 1	2) $\sqrt{3}:1$	3) 3: 1	4) $\sqrt{2}:1$	
28.	Two circular loops of	radii R and nR are r	nade of same wire. If	their M.I about	
	their normal axis thro	ough centre is in the r	atio 1: 8, the value of	n is	
	1) 6	2) 1	3) 2	4) 4	
29.	The radius of gyration	n of a solid sphere of i	radius 'R' about its ta	ngential axis is	
			_	_	
	$1) \frac{7R}{5}$	$2) \frac{3R}{5}$	3) $R\sqrt{\frac{7}{5}}$	4) $R\sqrt{\frac{3}{5}}$	
30.	The moment of inertia	a of an uniform circul	lar disc about its cent	ral axis is 'I'. Its	
	M.I about a tangent in its plane is equal to				
	1) 2I	2) 2.5 I	3) 1.5 I	$4) \; \frac{\mathrm{I}}{2}$	
31.	Two circular rings of	f equal mass and rac	dius are placed touch	ning each other.	
	The moment of inertia of the system about tangential axis in the plane of system				
	passing through point of contact of rings is				
	1) 3 mr <sup>2</sup>	2) $\frac{3}{2}$ mr <sup>2</sup>	3) 6 mr <sup>2</sup>	4) $\frac{5}{2}$ mr <sup>2</sup>	

32.	Four small spheres each of radius 'r' and mass 'm' are placed with their centers
	at the four corners of a square of side 'L'. The M.I. of the system about any side
	of square is

1) 
$$\frac{8mr^2}{5} + mL^2$$
 2)  $\frac{8mr^2}{5} + 2mL^2$  3)  $\frac{5mr^2}{8} + mL^2$  4)  $\frac{5mr^2}{8} + 2mL^2$ 

33. A disc of mass 'm' and radius R has a concentric hole of radius 'r'. It's M.I. about an axis through centre and normal to its plane is

1) 
$$\frac{m}{2}(R-r)^2$$
 2)  $\frac{m}{2}(R^2-r^2)$  3)  $\frac{m}{2}(R+r)^2$  4)  $\frac{m}{2}(R^2+r^2)$ 

34. A circular disc of radius 'R' and thickness R/6 has moment of inertia 'I' about an axis passing through its centre and perpendicular to its plane. It is melted and re-casted into a solid sphere. The M.I of the sphere about its diameter as axis of rotation is

35. A uniform circular disc of radius R lies in the XY - plane with its centre at the origin of co-ordinate system. Its moment of inertia about an axis, lying in the xy-plane, parallel to the x-axis and passing through a point on the y-axis at a distance y = 2R is  $I_1$ . Its moment of inertia about an axis lying in a plane perpendicular to xy-plane passing through a point on the x-axis at a distance x = d is  $I_2$ . If  $I_1 = I_2$ , the value of 'd' is

1) 
$$\frac{\sqrt{19}}{2}R$$
 2)  $\frac{\sqrt{17}}{2}R$  3)  $\frac{\sqrt{15}}{2}R$  4)  $\frac{\sqrt{13}}{2}R$ 

36. Three spheres, each of mass 'm' and radius 'R' are kept in touch with each other such that their centers from an equilateral triangle. The M.I. of the system about a median of triangle is

1) 
$$\frac{21}{5}MR^2$$
 2)  $\frac{16}{5}MR^2$  3)  $\frac{7}{2}MR^2$  4)  $\frac{4}{5}MR^2$ 

37. A mass is whirled in a circular path with an angular momentum L. If the length

	of string and angular velocity, both are doubled, the new angular momentum is				
	1) L	2) 4L	3) 8L	4) 16L	
38.	If 484 J of energy is spent in increasing the speed of a wheel from 60 rpm to 360				
	rpm, the M.I. of the w				
	1) $1.6 \text{ kg m}^2$	2) $0.3 \text{ kg m}^2$	3) $0.7 \text{ kg m}^2$	4) $1.2 \text{ kg m}^2$	
39.	The diameter of a rot	ating fly wheel is R.	Its coefficient of linea	r expansion is $\alpha$ .	
	If the temperature is increased by $\Delta T$ , the percentage change in its rotational				
	KE would be.				
	1) α.Δ <i>t</i> .100	2) 2.α. Δ <i>t</i> .×100	3) $\frac{\alpha \Delta t}{2} \times 100$	4) $\frac{2\alpha\Delta t}{5} \times 100$	
40.	The K.E. of a body ro	otating at 300 rpm is	62.8J. Its angular mo	mentum is	
	1) 1 Js	2) 2 Js	3) 4 Js	4) 8 Js	
41.	A thin circular ring	of mass 'm' and radi	us R is rotating abou	at its axis with a	
	constant angular velocity $\boldsymbol{\omega}$ . The objects each of mass M are attached gently to				
	him opposite ends of a diameter of the ring. The new angular velocity of ring is				
	1) $\frac{\omega m}{M+m}$	$2) \frac{\omega m}{m+2M}$	$3) \frac{\omega(m+2M)}{m}$	$4) \frac{\omega(m-2M)}{m+2M}$	
42.	A particle performs	uniform circular mot	ion with an angular	momentum L. If	
	the angular frequen	cy of particle is doub	oled and kinetic ener	gy is halved, its	
	angular momentum becomes				
	1) 4L	2) 2L	3) L/L	4) L/4	
43.	3. A heavy wheel of radius 20 cm and weight 10kg is to be dragged over a step of				
	height 10cm, by a horizontal force F applied at the centre of the wheel. The				
	minimum value of F is				
	1) 20 kgwt	2) 1 kgwt	3) $10\sqrt{3}kgwt$	4) $10\sqrt{2} kgwt$	

44. The M.I. of a uniform disc about an axis passing through its centre and perpendicular to its plane is  $1 \text{ kg m}^2$ . It is rotating with an angular velocity of  $100 \text{ rad s}^{-1}$ . A second disc of same mass and radius is joined to it coaxially. Now these two discs together continue to rotate about the same axis. Then the lose in kinetic energy in kilo joules is

1) 2.5

2) 3

3) 3.5

4) 4

45. A rod of length l is held vertically stationary with its lower end located at a point 'P', on the horizontal plane. When the rod is released to topple about 'P', the velocity of the upper end of the rod with which it hits the ground is

1)  $\sqrt{\frac{g}{\ell}}$ 

- 2)  $\sqrt{3g\ell}$
- 3)  $3\sqrt{\frac{g}{\ell}}$
- 4)  $\sqrt{\frac{3g}{\ell}}$  s

Kev

- 1) 4 2) 4 3) 3 4) 2 5) 3 6
  - 6) 2 7) 4
- 8) 2
- 9) 3 10) 4

- 11) 2 12) 1
- 13) 4
- 14) 3
- 16) 1
- 17) 1 18) 1
- 19) 4 20) 1

- 21) 1 22
- 23) 1
- 24) 4
- 25) 2

15)3

- 26) 3
- 27) 2
- 28) 3 2

38) 3

29) 3 30) 2

- 31) 1
- 32) 2 33
- 34)
- .
- 35) 3
- 36) 2
- 37) 3
- 39) 2 40) 3

- 41) 2 42) 4
- 43) 3
- 44) 1
- 45) 4

#### **Hints**

26. 
$$k^2 = k_{cg}^2 + r^2$$

$$5^2 = k_{cg}^2 + 4^2$$

$$k_c = 3cm$$

27. 
$$\frac{mr^2}{2} = \frac{mr^2}{4} + \frac{ml^2}{12}$$

$$\frac{l}{r} = \sqrt{3}/1$$

28. 
$$I = mr^2$$

but 
$$m = 2\pi r \times a$$

$$I \propto r^3$$

$$\frac{1}{8} = \frac{1}{r^3} \Rightarrow r = 2$$

29. 
$$I = \frac{7}{5}MR^2 = MK^2$$
  $K = R\sqrt{\frac{7}{5}}$ 

30. 
$$I = \frac{mr^2}{2} = \frac{5}{4}mr^2$$

$$I^1 = \frac{mr^2}{4} + mr^2 = 2.5 \text{ I}$$

$$I^{1} = \frac{mr^{2}}{4} + mr^{2} = 2.5 \text{ I}$$
31. 
$$I = \left[\frac{mr^{2}}{2} + mr^{2}\right] \times 2 = 3mr^{2}$$

32. 
$$I = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \left(\frac{2}{5}mr^2 + ml^2\right)^2$$

32. 
$$I = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \left(\frac{2}{5}mr^2 + ml^2\right)^2$$
  
33.  $I = \int_{r}^{R} dm.x^2 = \int_{r}^{R} \frac{M}{\pi(R^2 - r^2)} 2\pi x.dx.x^2$ 

$$I = \frac{M}{2} \left( R^2 - r^2 \right)$$

34. 
$$I = \frac{MR^2}{2}$$
  $I^1 = \frac{2}{5}M.x^2$  Where  $x = R/2$ 

$$=\frac{2}{5}M.\frac{R^2}{4} \qquad \qquad =\frac{I}{5}$$

35. 
$$\frac{MR^2}{4} + M.4R^2 = I_1 \frac{MR^2}{2} + Md^2 = I_2$$

$$\frac{MR^2}{2} + Md^2 = \frac{MR^2}{4} + 4MR^2$$

$$d^2 = \frac{17R^2}{4} - \frac{R^2}{2} \Rightarrow d = \frac{R\sqrt{15}}{2}$$

$$\frac{MR^{2}}{2} + Md^{2} = \frac{MR^{2}}{4} + 4MR^{2}$$

$$d^{2} = \frac{17R^{2}}{4} - \frac{R^{2}}{2} \Rightarrow d = \frac{R\sqrt{15}}{2}$$

$$36. \quad I = \frac{2}{5}mr^{2} + \left(\frac{7}{5}mr^{2}\right) \times 2 = \frac{16}{5}mr^{2}$$

$$37. \quad L = I\omega = mr^{2}. \omega$$

$$L \alpha r^{2}\omega$$

$$L^{1} = 8 L$$

$$38. \quad 484 = \frac{1}{2}I\left[\left(12\pi\right)^{2} - \left(2\pi\right)^{2}\right]$$

$$484 = \frac{1}{2}I \times 14\pi \times 10\pi$$

37. 
$$L = I\omega = mr^2$$
.  $\omega$ 

$$L \alpha r^2 \omega$$

$$L^1 = 8 L$$

38. 
$$484 = \frac{1}{2}I[(12\pi)^2 - (2\pi)^2]$$

$$484 = \frac{1}{2}I \times 14\pi \times 10\pi$$

$$I = 0.7 kgm^2$$

38. 
$$484 = \frac{1}{2}I[(12\pi)^2 - (2\pi)^2]$$
  
 $484 = \frac{1}{2}I \times 14\pi \times 10\pi$   
 $I = 0.7kgm^2$   
39.  $E = \frac{L^2}{2I} \implies E\alpha \frac{1}{I} = \frac{\Delta E}{E} = -\frac{\Delta I}{I}$ 

$$\frac{\Delta E}{E} = 2 \alpha \Delta t \qquad \frac{\Delta E}{E} \times 100 = 2 \alpha \Delta t \times 100$$

40. 
$$E = \frac{L^2}{2I} = \frac{1}{2}L.\omega$$
  $L = \frac{2E}{\omega} = \frac{2 \times 62.8}{10 \times 3.14} = 4$ 

41. 
$$mR^2 \omega = R^2 (m+2M) \omega^1$$

$$\omega^1 = \frac{m}{m + 2M} \ \omega$$

42. 
$$E = \frac{1}{2} LW$$
 OR  $E \propto LW$ 

OR 
$$E \propto LW$$

$$\frac{E}{2} \propto L^1 \ 2W$$

$$\frac{E}{2} \propto L^1 \ 2W$$
 OR  $2 = \frac{L}{L^1} \times \frac{1}{2}$ 

$$\frac{L}{I_{\cdot}^{1}}=4$$

$$\Rightarrow L^1 = L/4$$

43. 
$$F \times 10 = W \sqrt{20^2 - 10^2}$$

$$F \times 10 = W \times 10\sqrt{3}$$

$$F = W \sqrt{3}$$

$$=10\sqrt{3} kgwt$$

44. 
$$I\omega = (I_1 + I_2)\omega$$

$$I \times 100 = 2I$$
.

$$\omega^1 = 25 \ rads^{-1}$$

$$\frac{L}{L^{1}} = 4 \qquad \Rightarrow L^{1} = L/4$$
43.  $F \times 10 = W \sqrt{20^{2} - 10^{2}} \qquad F \times 10 = W \times 10\sqrt{3}$ 

$$F = W \sqrt{3} \qquad = 10\sqrt{3} \ kgwt$$
44.  $I\omega = (I_{1} + I_{2}) \omega \qquad \text{I x } 100 = 2\text{I}.$ 

$$\omega^{1} = 25 \ rads^{-1}$$

$$\Delta E = \frac{1}{2} \times \frac{1 \times 1}{2} \times 100^{2}$$
45.  $mg \frac{1}{2} = \frac{1}{2} \frac{ml^{2}}{3} \omega^{2} \qquad \omega = \sqrt{\frac{3g}{l}}$ 

45. 
$$mg \frac{l}{2} = \frac{1}{2} \frac{ml^2}{3} \omega$$

$$\omega = \sqrt{\frac{3g}{l}}$$