COORDINATE SYSTEM

2D GEOMETRY

SYNOPSIS

1. The distance between the point A(x_1, y_1) and B(x_2, y_2) is AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
2. Three or more points are said to be collinear if they lie in a line.
3. If A, B and C are collinear, then AB + BC = AC or AC + CB = AB or BA + AC = BC.
4. The point which divides the line segment joining the points A(x_1, y_1) B(x_2, y_2) in the ratio l : m is P$\left(\frac{lx_2+mx_1}{l+m},\frac{ly_2+my_1}{l+m}\right)$(l + m ¹ 0).
   Note (1) : lm > 0 then P lies internally and lm < 0 then P lies externally.
   Note (2) : If P divides AB externally in the ratio l : m, then P lies on AB produced if |m|< |l| and on BA produced if |m| > |l|.
5. The mid point of the line segment joining A (x_1, y_1) B(x_2, y_2) is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.
6. Let A, B be two points. The points which divide $\overline{AB}$ in the ratio 1 : 2 and 2 : 1 are called points of trisection of $\overline{AB}$.
7. If P(x, y) lies in the line joining A(x_1, y_1) B(x_2, y_2) then $\frac{x_1-x}{x-x_2}=\frac{y_1-y}{y-y_2}$ and P divides in the ratio $x_1 - x : x - x_2$ or $y_1 - y : y - y_2$.
8. x-axis divides the line segment joining (x_1, y_1) (x_2, y_2) in the ratio $- y_1 : y_2$.
9. y-axis divides the line segment joining in the ratio $- x_1 : x_2$.
   If D(a_1, b_1), E(a_2, b_2) F(a_3, b_3) are the mid points of the sides , , of DABC then
   A = ($a_2 + a_3 - a_1$, $b_2 + b_3 - b_1$).
   B = ($a_3 + a_1 - a_2$, $b_3 + b_1 - b_2$).
   C = ($a_1 + a_2 - a_3$, $b_1 + b_2 - b_3$).
10. (a) If (a_1, b_1), (a_2, b_2), (a_3, b_3) are three vertices of a parallelogram in order then fourth vertex is ($a_1 + a_3 - a_2$, $b_1 + b_3 - b_2$).
    Note : If three vertices of parallelogram are given and if order is not given then we get three points as fourth vertex.
    i.e., if three non-collinear points are given then we get three parallelograms with these three as vertices of parallelogram.
    Note (1) : If D is midpoint of BC of triangle ABC then $AB^2 + AC^2 = 2(AD^2 + BD^2)$.
    (2) Internal angular bisector of angle A of DABC divides the opposite side BC in the ratio AB : AC.
    (3) If a, b, c are lengths of sides BC, CA, AB of DABC and if I is incentre then I divides the internal angular bisector AD in the ratio b + c : a.
    (4) Finding the fourth vertex of a quadrilateral when the order of 3 vertices is not given
    (i) Rhombus or Square : If A, B, C are the 3 points such that AB = AC then the fourth vertex of the rhombus is the vertex of the parallelogram opposite to A.

(ii) Rectangle : The fourth vertex is the vertex opposite to the vertex where right angle forms. If G is centroid of DABC then

(i) $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

(ii) Area of DGAB = Area of DGBC = Area of DGCA = 1/3 Area of DABC.

If D, E, F are the mid points of sides BC, CA, AB of DABC then

Area DDEF = Area of DAFE = Area of DBDF = Area of triangle CED = ½

Area of each parallelogram formed = ¼ Area of DABC.

In a triangle ABC if BC is the largest side then

(i) $AB^2 + AC^2 = BC^2$ then triangle ABC is right angled.

(ii) $AB^2 + AC^2 > BC^2$ then triangle ABC is Acute angled triangle.

(iii) $AB^2 + AC^2 < BC^2$ then triangle ABC is Obtuse angled triangle.

Note : In the above 3 cases if AB = AC then the triangle is isosceles also.

11. Harmonic conjugate points : If P and Q divide AB internally and externally in the same ratio, then P is called as harmonic conjugate of Q and Q is called as harmonic conjugate of P, also P, Q are a pair of conjugate points w.r.t. A and B.

Note : (1) If P, Q are harmonic conjugate points w.r.t A, B then A, B are harmonic conjugate points w.r.t. P, Q.

(2) If P, Q divide AB in the ratio l : m internally and externally then A, B divide PQ in the ratio (l – m) : (l + m).

12. Let G be the centroid of ΔABC then

i) $G = \dfrac{A+B+C}{3}$ ii) $C = 3G – A – B$

13. Let G be the centroid ΔABC and D, E, F be the midpoints of BC, CA, AB respectively, then G divides the medians AD, BE, CF in the ratio 2 : 1.

14. Let D, E, F be the midpoints of the sides of ΔABC then centriod of ΔDEF = centriod of ΔABC.

15. The orthocentre of a right angled triangle is the vertex at the right angle.

16. The circumcentre of a right angled triangle is the midpoint of the hypotenuse.

17. For right angled triangle, circum radius = $\dfrac{\text{hypotenuse}}{2}$.

18. Orthocentre O, circumcentre S, centriod G of a triangle are collinear, G divides $\overline{OS}$ in the ratio 2:1. Then

$$G = \frac{2S+O}{3}, S = \frac{3G-O}{2}, O = 3G - 2S$$

19. If O is the orthocentre of ΔABC then A, B, C are the orthocentres of $\Delta OBC, \Delta OCA, \Delta OAB$ respectively.

20. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of ΔABC and if $a = BC$, $b = CA$, $c = AB$ then incentre,

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Excentre opposite to A is $$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

Excentre opposite to B is $$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right)$$

Excentre opposite to C is $$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

21. Internal angular bisector of angle A of ΔABC divides the opposite side BC in the ratio $AB : AC$.
22. If a, b, c are lengths of sides BC, CA, AB of ΔABC and if I is incentre then I divides the internal angular bisector AD in the ratio $b+c:a$
23. If I is the incentre and I_1, I_2, I_3 are the excentres of ΔABC then, I is the orthocentre of $\Delta I_1I_2I_3$.
24. The area of the triangle formed by the points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is
$$\frac{1}{2}\left|\Sigma x_1(y_2-y_3)\right| \text{ or } \frac{1}{2}\left|\Sigma(x_1y_2-x_2y_1)\right| \text{ or } \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix} \text{ (or) } \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$
25. The area of the triangle formed by the points $O(0,0)$, $A(x_1,y_1)$, $B(x_2,y_2)$ is $\frac{1}{2}\left|x_1y_2 - x_2y_1\right|$.
26. The area of the quadrilateral formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, $D(x_4, y_4)$ taken in order is $\frac{1}{2}\begin{vmatrix} x_1-x_3 & x_2-x_4 \\ y_1-y_3 & y_2-y_4 \end{vmatrix}$.
27. If G is centroid of ΔABC then,

    Area of ΔGAB = Area of ΔGBC = Area of $\Delta GCA = \frac{1}{3}$ Area of ΔABC.
28. If D, E, F are the mid points of sides BC, CA, AB of ΔABC then

    area ΔDEF = Area of ΔAFE = Area of ΔBDF = Area of $\Delta CED = \frac{1}{4}$ Area of ΔABC.
29. If D is the midpoint of BC of a triangle ABC then $AB^2 + AC^2 = 2(AD^2 + BD^2)$.
30. In a ΔABC, if $BC=a$, $CA=b$, $AB=c$ then
    i) the length of median through A
    $$= \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$
    ii) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
31. If G is centroid of ΔABC and D, E, F are the mid points of the sides BC, CA, AB respectively then $AB^2 + BC^2 + CA^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2) = 3(GA^2 + GB^2 + GC^2)$
32. Let D, E, F be the feet of the altitudes and X, Y, Z be the mid points of the sides of triangle ABC. Let P, Q, R are the mid points of AO, BO, CO where 'O' is the orthocentre of the triangle. Then D, E, F ; X, Y, Z ; P, Q, R lie on a circle called nine point circle of the triangle. The centre of the nine point circle (denoted by N) is the midpoint of the line segment joining the ortho centre and circum centre ie., $ON = NS$. Radius of the nine point circle = $R/2$.

33. In a triangle ABC,

$OG : GS = 2 : 1 ON : NG : GS = 3 : 1 : 2$

Here G = centroid, O = orthocentre,

S = circum centre of ΔABC, N = centre of nine point circle.

34. The orthocentre of the triangle with vertices $(0,0)$, (x_1, y_1) (x_2, y_2) is

$$= (k(y_2 - y_1), k(x_1 - x_2)) \text{ where } k = \frac{x_1x_2 + y_1y_2}{x_2y_1 - x_1y_2}$$