COORDINATE SYSTEM

2D GEOMETRY

OBJECTIVE QUESTIONS

1. If the distance between the points $(a\cos\theta, a\sin\theta)$ and $(a\cos\phi, a\sin\phi)$ is 2a, then $\theta =$

   1) $2n\pi + \pi + \phi, n \in Z$ 2) $n\pi + \frac{\pi}{2} + \phi, n \in Z$

   3) $n\pi - \phi, n \in Z$ 4) $2n\pi + \phi, n \in Z$

2. If $A = (ar^2, 2at)$, $B = \left(\frac{a}{t^2}, -\frac{2a}{t}\right)$, $S(a,0)$ then $\frac{1}{SA} + \frac{1}{AB} =$

   1) a 2) 1/a 3) 2/a 4) 2a/3

3. The three points (2,–4), (4,–2), (7,1)

   1) are collinear

   2) from an equilateral triangle

   3) form a right angled triangle

   4) form an isosceles triangle

4. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

   1) lie on an ellipse 2) lie on a circle

   3) are vertices of a triangle 4) lie on a straight line

5. If (2,4), (2,6) are two vertices of an equilateral triangle then the third vertex is

   1) $(2+\sqrt{3}, 5)$ 2) $(\sqrt{3}-2, 5)$ 3) $(5, 2+\sqrt{3})$ 4) $(5, 2-\sqrt{3})$

6. If (3,4), (–2,3) are two vertices of an equilateral triangle then the third vertex is

   1) $\left((1+\sqrt{3})/2, (7+5\sqrt{3})/2\right)$ 2) $\left((1-\sqrt{3})/2, (7-5\sqrt{3})/2\right)$

   3) $\left((1-\sqrt{3})/2, (7+5\sqrt{3})/2\right)$ 4) none

7. If (2,4), (4,2) are the extremities of the hypotenuse of a right angled isosceles triangle, then the third vertex is .

   1)(2,2) or (4,4) 2) (3,3) or (4,4)

   3) (2,2) or (3,3) 4) (2,3) or (3,2)

8. If 0 is the origin and if A$((x_1, y_1), B(x_2, y_2)$ are two points then OA.OBcos$\angle$AOB= thenthen the third vertex is .

1)(2,2) or (4,4) 2) (3,3) or (4,4)

3) (2,2) or (3,3) 4) (2,3) or (3,2)

9. If 0 is the origin and $A(x_1, y_1), B(x_2 y_2)$ are two points then OA.OB.sin$\angle$AOB =

1) $x_1^2 + y_1^2 - x_2^2 - y_2^2$ 2) $x_1x_2 + y_1y_2$

3) $x_1y_2 + x_2y_1$ 4) $[x_1y_2 - x_2y_1]$

10. If the vertices of a triangle A,B,C are A(0,0), B(2,1),C(9,–2) then cos B=

1) $\frac{16}{5\sqrt{17}}$ 2) $\frac{11}{\sqrt{290}}$ 3) $\frac{16}{5\sqrt{7}}$ 4) $\frac{-11}{\sqrt{290}}$

11. The points (–5,12), (–2,-3),(9,–10), (6,5) taken in order, form

1) Parallelogram 2) rectangle

3) rhombus 4) square

12. The points (–a, –b), (0,0), (a,b), (a^2, ab) are

1) collinear 2) vertices of a parallelogram

3) concyclic

4) vertices of a rectangle

13. The points which divide internally and externally the line segment joining the points (1,7), (6,-3) in the ratio 2:3 are

1) (3,3), (15,15) 2) (3,3), (–15,–15)

3) (3,3), (–9,27) 4) (–3,–3), (9,27)

14. The fourth vertex of the rectangle whose other vertices are (4,1), (7,4), (13,–2) is

1) (10,–5) 2) (10,5) 3) (–10,5) 4)(–10,–5)

15. If (2,1), (-2,5) are two opposite vertices of a square then the area of the square is

1) 4 2) 12 3) 16 4) 36

16. The ratio in which (2,3) divides the line segment joining (4,8), (-2,-7) is

1) 2:1 externally 2) 2:3

3) 4:3 externally 4) 1:2

17. If Q is the harmonic conjugate of P w.r.t A,B and AP=2, AQ =6 then AB=

1) 5 2) 1 3) 3 4) 2

18. P=(–5,4) and Q=(–2,–3), if $\overline{PQ}$ is produced to R such that P divides $\overline{QR}$ externally in the ratio 1:2, then R is

1) (1,10) 2) (1,–10) 3)(10,1) 4)(2,–10)

19. A(a,b) and B(0,0) are two fixed points. M_1 is the mid point of $\overline{AB}$ M_2 is the midpoint of AM_1, M_3 is the midpoint of $\overline{AM_2}$ and so on. Then M_5 is

1) $\left(\frac{7a}{8},\frac{7b}{8}\right)$ 2) $\left(\frac{15a}{16},\frac{15b}{16}\right)$

3) $\left(\frac{31a}{32},\frac{31b}{32}\right)$ 4) $\left(\frac{63a}{64},\frac{63b}{64}\right)$

20. If the point $\left(x_1+t\left[x_2-x_1\right],y_1+t\left[y_2-y_1\right]\right)$ divides the joint of $\left(x_1,y_1\right)$ and $\left(x_2,y_2\right)$ internally, then

1) t<0 2)0<t<1 3)t >1 4) t=1

21. The points D,E,F are the midpoints of the sides $\overline{BC},\overline{CA},\overline{AB}$ of ΔABC rspectively. If A= (–2,3), D=(1,–4), E=(–5,2), then F =

1) (4,3) 2) (4,–3) 3)(–4,3) 4)(–4,–3)

22. The centroid of a triangle is (2,3) and two of its vertices are (5,6) and (–1,4). The third vertex of the triangle is

1) (2,1) 2) (2,–1) 3) (1,2) 4) (1,–2)

23. If a vertex of a triangle is (1,1) and the midpoints of two sides through this vertex are (–1,2) and (3,2), then the centroid of the triangle is

1) $\left(-1,\frac{7}{3}\right)$ 2) $\left(\frac{-1}{3},\frac{7}{3}\right)$ 3) $\left(1,\frac{7}{3}\right)$ 4) $\left(\frac{1}{3},\frac{7}{3}\right)$

24. The point P is equidistant from A(1,3), B(–3,5) and C(5,–1). Then PA=

1) 5 2) $5\sqrt{5}$ 3) 25 4) $5\sqrt{10}$

25. The vertices of a triangle are (6,6), (0,6). The distance between its circumcentre and centroid is

1) $2\sqrt{2}$ 2) 2 3) $\sqrt{2}$ 4) 1

26. The incentre of the triangle formed by the points (0,0), (5,12), (16,12) is

1) 6,9) 2) (7,9) 3) (6,7) 4) (9,7)

27. If $\ell_1\ell_2\ell_3$ are excentres of the triangle with vertices (0,0), (5,12), (16,12) then the orthocentre of $\Delta\ell_1\ell_2\ell_3$ is

1) (6,9) 2) (7,9) 3) (6,7) 4) (9,7)

28. If the orthocentre and circumcentre of a triangle are (–3,5), (6,2) then the centroid is

1) (2,–3) 2) (3,3) 3) (4,3) 4) (1,–3)

29. If (3,–2) is the orthocentre and (–1,4) is the circumcentre of ΔABC then ninepoint centre of ΔABC is

1) (2,3) 2) (1,1) 3) (3,2) 4)(–1,2)

30. The area of the triangle with vertices at (–4,–1),(1,2), (4,–3) is

1) 12 2) 18 3) 17 4) 30

31. The area of the triangle formed by the points (a,1/a), (b,1/b), (c,1/c) is

1) $\left|\frac{(a+b)(b+c)(c+a)}{2abc}\right|$ 2) $\left|\frac{(a-b)(b-c)(c-a)}{2abc}\right|$

3) $\left|\frac{(a+b)(b-c)(c-a)}{2abc}\right|$ 4) $\left|\frac{(a+b)(b-c)(c+a)}{2abc}\right|$

32. The area of the triangle with vertices (a,b), (ar,bs), (ar^2,bs^2) is

1) $|ab(r-1)(s-1)|$ 2) $|ab(r-1)(s-1)(s-r)|$

3) $\frac{1}{2}|ab(r+1)(s+1)(s-r)|$ 4) $\frac{1}{2}|ab(r-1)(s-1)(s-r)|$

33. If A=(–3,4), B(–1,–2),C(5,6),D(x,–4) are the vertices of a quadrilateral such that area of Δ ABD = 2[Area of Δ ACD] then x =

1) 6 2) 9 3) 69 4) 96

34. The point A divides the join of P(–5,1) and Q(3,5) in the ratio k:1. The values of k for which the area of ΔABC where B(1,5), C (7,–2) is 2 sq. units is

1)7,31/9 2) –7,31/9

3) 7,–31/9 4) –7,–31/9

35. Let A(h,k), B(1,1) and C(2,1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which k can take is given by

1) (1,3) 2) (0,2) 3) (–1,3) 4)(–3,–2)

36. If (k,2–2k),(–k+1,2k),(–4–k,–6–2k), are collinear, then k =

1) 2 2) 5 3) 1/2–1 4) –1/2,2

37. A string of length 12 is bent first into a square PQRS and then into a right angled triangle PQT by keeping the side PQ of the square fixed. Then the area of PQRS =

1) Area of Δ PQT 2) $\frac{3}{2}$(Area of Δ PQT)

3) 2(Area if Δ PQT) 4) none

38. Instead of walking along two adjacent sides of a rectangular field, aboy took a short cut along the diagonal and saved distance equal to half the longer side. Then the ratio of the shorter side to the longer side is .

1) 1:2 2) 2:3 3) 1:4 4) 3:4

39. S_1 and S_2 are the inscribed and circumscribed circles of a triangle with sides 3,4 and 5. Then (area of S_1)/(area of S_2) =

1) 16/25 2) 4/25 3) 9/25 4) 9/16

40. If the area of the triangle whose vertices are (b,c) (c,a) (a,b) is p then the area of the triangle whose vertices $(ac-b^2, ab-c^2)$ $(ab-c^2, bc-a^2)$ and $(bc-a^2, ac-b^2)$ is

1) $(a+b+c)^2$ 2) P(a+b+c)

3) $P(a+b+c)^2$ 4)none

41. ABC is an isosceles triangle with side AB=AC. If the coordinates of the base are B(a+b,b–a) and C(a–b,a+b), $a \neq b \neq 0$, then the coordinates of the vertex A can be

1) (a,b) 2) (b,a)

3) (a/b,b/a) 4) (b/a,a/b)

42. If a, x_1, x_2 are in G.P. with common ratio r, and b, y_1, y_2 are in G.P. with common ratio s where $s-r=2$, then the area of the triangle with vertices (a,b), (x_1,y_1), (x_2,y_2) is

1) $\left|ab\left(r^2-1\right)\right|$ 2) $ab\left(r^2-s^2\right)$

3) $ab\left(s^2-1\right)$ 4) abrs

43. ABC is an isosceles triangle of area $\frac{25}{6}$ sq. unit if the coordinates of base are B(1,3) and C(–2,7), the coordinates of A are

1) $(1,6)\left(-\frac{11}{6},\frac{5}{6}\right)$ 2) $\left(-\frac{1}{2},5\right),\left(4,\frac{5}{6}\right)$ 3) $\left(\frac{5}{6},6\right),\left(-\frac{11}{6},4\right)$ 4) $\left(5,\frac{5}{6}\right),\left(\frac{11}{6},4\right)$

44. If A and B are two points having coordinates (3,4) and (5,–2) respectively and P is a point such that PA= PB and area of triangle PAB =10 square unit, then the coordinates of P are

1) (7,4) or (13,2) 2) (7,2) or (1,0)

3) (2,7) or (4,13) 4) none of these

COORDINATE SYSTEM

2D GEOMETRY

SOLUTIONS

1. Ans.1

Sol:Distance $2a \Rightarrow (a\cos\theta - a\cos\phi)^2 + (a\sin\theta - a\sin\phi)^2 = 4a^2$

$$\Rightarrow (\cos\theta - \cos\phi)^2 + (\sin\theta - \sin\phi)^2 = 4$$

$$\Rightarrow 2 - 2\cos(\theta-\phi) = 4 \;\Rightarrow \cos(\theta-\phi) = -1$$

$$\Rightarrow \theta - \phi = 2n\pi \pm \pi \Rightarrow \theta = 2n\pi \pm \pi + \phi, n \in Z.$$

2. Ans.2

Sol: $SA = \sqrt{[a(r^2-1) + 4a^2t^2}$

$$= \sqrt{a^2\left[(r^2-1) + 4r^2\right]}$$

$$= \sqrt{a^2(r^2+1)^2} = a(t^2+1)$$

$$SB = \sqrt{[a\left(\frac{1}{t^2}-1\right) + \left(\frac{2a}{t}\right)^2} \quad \sqrt{a^2\left[\left(\frac{1}{t^2}-1\right)+\frac{4}{t^2}\right]} = \sqrt{a^2(r^2+1)^2}$$

$$= \sqrt{a^2\left(\frac{1}{t^2}+1\right)^2} = a\left(\frac{1}{t^2}+1\right)$$

$$\frac{1}{SA} + \frac{1}{SB} = \frac{1}{a(t^2+1)} + \frac{1}{a(1/t^2+1)}$$

$$= \frac{1}{a(t^2+1)} + \frac{t^2}{a(t^2+1)} = \frac{1+t^2}{a(1+t^2)} = \frac{1}{a}$$

3. Ans.1

Sol:A(2,–4);B(4,–2);C(7,1)

AB= $\sqrt{(2-4)^2 + (-4+2)^2} = 2\sqrt{2},$

BC $= \sqrt{(4-7)^2 + (-2-1)^2} = 2\sqrt{3},$

CA $= \sqrt{(2-7)^2 + (-4-1)^2} = 2\sqrt{5},$

$AB^2 + BC^2 = 8 + 12 = 20 = AC^2 \cdot$

$\therefore$ A,B,C are collinear

4. Ans.4

Sol: $x_1 = a, x_2 = ar, x_3 = ar^2$ and $y_1 = b, y_2 = br, y_3 = br^2$ then area of Δ whose vertices are

$(x_1, y_1)(x_2 y_2)$ and $(x_3 y_3)$ is given by $\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ a & ar & ar^2 \\ b & br & br^2 \end{vmatrix}$

$$= \frac{1}{2}ab\begin{vmatrix} 1 & 1 & 1 \\ 1 & r & r^2 \\ 1 & r & r^2 \end{vmatrix} = 0$$

$\therefore$ $(x_1 y_1), (x_2 y_2)$ and (x_3, y_3) are collinear i.e. lie on straight line.

5. Ans.1

Sol: Third Vertex $= \left(\frac{2 + 2 \pm \sqrt{3}(4-6)}{2} \frac{4 + 6 + \sqrt{3}(2-2)}{2}\right) = \left(2 + \sqrt{3}, 5\right)$

6.Ans.3

Sol:Third Vertex $= \left(\frac{3 - 2 \pm \sqrt{3}(4-3)}{2}, \frac{4 + 3 + \sqrt{3}(3+2)}{2}\right)$

$$= \left(\frac{1 \pm \sqrt{3}}{2}, \frac{7 + 5\sqrt{3}}{2}\right)$$

$$= \left(\frac{1+\sqrt{3}}{2} \frac{3 - 5\sqrt{3}}{2}\right), \left(\frac{1-\sqrt{3}}{2}, \frac{7 + 5\sqrt{3}}{2}\right)$$

7. Ans.1

Sol: Third Vertex $= \left(\frac{2 + 4 \pm (4-2)}{2}, \frac{(4+2) + (2-4)}{2}\right)$

$$= (3 \pm 1, 3 \pm 1) = (4,4) \text{ or } (2,2)$$

8. Ans.3

Sol:OA.OB $\cos\angle AOB = \frac{1}{2}\left(OA^2 + OB^2 - AB^2\right) = \frac{1}{2}$

$$\left[\left(x_1^2 + y_1^2\right) + \left(x_2^2 + y_2^2\right) - \left\{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2^2\right)\right\}\right]$$

$= \frac{1}{2}\left[x_1^2+y_1^2+x_2^2+y_2^2-x_1^2-x_2^2+2x_1x_2-y_1^2+y_2^2+2y_1y_2\right] \quad = x_1x_2+y_1y_2$

9.Ans.4

Sol: OA.OB $\sin\angle AOB = 2$(Area of ΔOAB)

$= |x_1y_2 - y_2y_1|$.

10.Ans.4

Sol: a=BC= $\sqrt{49+9}=\sqrt{58}, b=CA=\sqrt{81+4}=\sqrt{85}, c$

$= AB = \sqrt{4+1}=\sqrt{5} \quad \cos B = \frac{c^2+a^2-b^2}{2ca}$

$= \frac{5+58-85}{2\times\sqrt{5}\times\sqrt{58}} = \frac{-22}{2\sqrt{290}} = \frac{-11}{\sqrt{290}}$

11. Ans.1

Sol: $A(-5,12), B(-2,-3), C(9-10), D(6,5)$

$AB=\sqrt{9+225}=\sqrt{234}, BC=\sqrt{121+49}$

$=\sqrt{170}, AC=\sqrt{196+484}=\sqrt{680}$

12. Ans.1

Sol: $A(0,0), B(-a,-b), C(a,b), D(a^2,ab)$

Slope of AB = b/a, Slope of AC = b/a, Slope of AD = ab/a^2= b/a. $\therefore$ A,B,C,D are collinear.

13. Ans.3

Sol: Internally is $\left(\frac{2(6)+3(1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right) = (3,3)$.

Externally is $\left(\frac{2(6)-3(1)}{2-3}, \frac{2(-3)-3(7)}{2-3}\right)$

= (–9,27).

14.Ans.1

Sol: (4–7+13,1–4–2) = (10–5)

15.Ans.3

Sol: Diagonal $p=\sqrt{(2+2)^2+(1-5)^2}=4\sqrt{2}$, Area of the square = $p^2/2$ = 16.

16. Ans.4

Sol: (4–2): (2+2)= 1:2

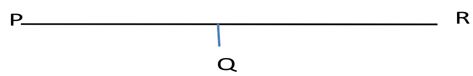
17. Ans.3

Sol: AP, AB, AQ are in H.P $\Rightarrow AB = \dfrac{2AP.AQ}{AP + AQ}$

$$= \frac{2(2)(6)}{2+6} = 3$$

18. Ans.2

Sol: p divides QR in the ratio 1:2 externally $\Rightarrow$ PQ:PR = 1:2 $\Rightarrow$ 2PQ = PR $\Rightarrow$ Q is the midpoint of PR $\Rightarrow$ R = 2Q–p=(1,10) .



19. Ans. 3

Sol: $M_1 = (a/2, b/2), M_2 = (3a/4, 3b/4)$, $M_3 = (7a/8, 7b/8), M_4 = (15a/16, 15b/16)$,

$M_5 = (31a/32, 31b/32)$

20. Ans.2

Sol: $(x_1 + t[x_2 - x_1], y_1 + t[y_2 - y_1])$

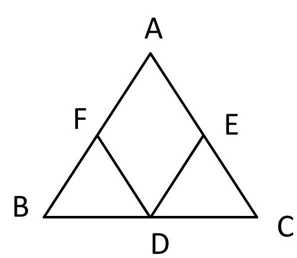
$= ([1-t]x_1 + tx_2, [1-t]y_1 + ty_2)$ divides the join of $(x_1, y_1), (x_2, y_2)$ internally

$\Rightarrow 1 - t > 0, t > 0 \Rightarrow 0 < t < 1$

21. Ans.2

Sol: A = E+F–D

$\Rightarrow$ F = A + D – E = (–2+1+5,3–4–2) = (4–3)

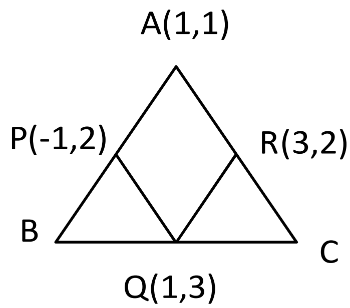


22. Ans.2

Sol: Third vertex C=3G–A–B=3(2,3)–(5,6)–(–1,4) = (2,–1)

23. Ans.3

Sol: Centroid of Δ ABC = centroid of Δ PQR = $\left(\frac{-1+1+3}{2}, \frac{2+3+2}{3}\right) = \left(1, \frac{7}{3}\right)$



24. Ans.4

Sol: PA=PB=PC $\Rightarrow$

$-2\alpha - 6\beta + 10 = 6\alpha - 10\beta + 34 = -10\beta + 2\beta + 26$

$\Rightarrow 8\alpha - 4\beta + 24 = 0, 16\alpha - 12\beta + 8 = 0$

$\Rightarrow \alpha = -8, \beta = -10$

$\therefore$ PA = $\sqrt{(1+8)^2 + (3+10)^2} = \sqrt{81+169}$

$= \sqrt{250} = 5\sqrt{10}$

25. Ans.3

Sol: A(6,6), B(0,6), C(6,0), $\angle A = 90°$.

Centroid of Δ ABC is $G\left(\frac{6+6}{3}, \frac{6+6}{3}\right) = (4,4)$. Circumcentre

$S = \frac{B+C}{2} = (3,3)$

Distance between circumcentre, centroid

$= \sqrt{(4-3)^2 + (4-3)^2} = \sqrt{1+1} = \sqrt{2}$.

26. Ans.2

Sol: A(0,0), B(5,12), C(16,12) a = BC = 11, b=CA= 20, c=AB =13

Incentre =

$\left(\frac{11(0)+20(5)+13(16)}{11+20+13}, \frac{11(0)+20(12)+13(12)}{11+20+13}\right)$

$= \left(\frac{308}{44}, \frac{396}{44}\right) = (7,9)$

27. Ans.2

Sol: A=(0,0), B(5,12), C=(16,12) $\Rightarrow$ a = 11, b = 20, c = 13

orthocentre of $\Delta\ell_1\ell_2\ell_3$ = Incentre of Δ ABC

$$\left(\frac{11(0)+20(5)+13(16)}{11+20+13}, \frac{11(0)+20(12)+13(12)}{11+20+13}\right)$$

= (7,9).

28. Ans.2

Sol: P(–3,5), S(6,2).

$$\therefore\ G = \left(\frac{2(6)+1(-3)}{2+1}, \frac{2(2)+1(5)}{2+1}\right) = (3,3)$$

29. Ans.2

Sol: Orthocentre H = (3,–2), circumcentre S=(–1,4) Ninepoint centre N = Midpoint of HS = (1,1)

30. Ans.3

$$\text{Sol: Area} = \frac{1}{2}\begin{vmatrix}(-4-1) & (-1-2)\\(-4-4) & (-1+3)\end{vmatrix} = \frac{1}{2}\begin{vmatrix}-5 & -3\\-8 & 2\end{vmatrix} = \frac{1}{2}\left|-10-24\right| = 17 \text{ sq. units.}$$

31. Ans.2

$$\text{Sol: Area} = = \frac{1}{2}\begin{vmatrix}(a-b) & \left(\frac{1}{a}-\frac{1}{b}\right)\\(a-c) & \left(\frac{1}{a}-\frac{1}{c}\right)\end{vmatrix} = \frac{1}{2}$$

$$\begin{vmatrix}(a-b) & \frac{-(a-b)}{ab}\\-(c-a) & \frac{(c-a)}{ca}\end{vmatrix} = \frac{1}{2}\left|\frac{(a-b)(c-a)}{ca} - \frac{(a-b)(c-a)}{ab}\right|$$

$$= \frac{1}{2}\left|\frac{(a-b)(c-a)}{abc}(b-c)\right| = \frac{1}{2}\left|\frac{(a-b)(b-c)(c-a)}{abc}\right|$$

32. Ans.4

$$\text{Sol: Area} = \frac{1}{2}\begin{vmatrix}a-ar & a-ar^2\\b-bs & b-bs^2\end{vmatrix} = \frac{1}{2}\left|a(1-r)(1-s)[1+s-1-r]\right| = \frac{1}{2}\left|ab(r-1)(s-1)(s-r)\right|$$

33. Ans.3

Sol: $\frac{1}{2}\begin{vmatrix} -2 & 6 \\ -(x+3) & 8 \end{vmatrix} = 2\times\frac{1}{2}\begin{vmatrix} -8 & -2 \\ -(x+3) & 8 \end{vmatrix}$

$\Rightarrow |6(x+3)-16| = 2|-64-2(x+3)|$

$\Rightarrow 2|3x+1| = 2|-2x-70| \Rightarrow 3x+1 = \pm(2x+70) \Rightarrow x = 69; 71/5$

34. Ans.1

Sol: $A = \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$

Area of Δ ABC $= 2 \Rightarrow \frac{1}{2}\begin{vmatrix} 1-7 & 5+2 \\ 1-\frac{3k-5}{k+1} & 5-\frac{5k+1}{k+1} \end{vmatrix} = 2 \Rightarrow \begin{vmatrix} -6 & 7 \\ \frac{-(2k-6)}{k+1} & \frac{4}{k+1} \end{vmatrix} = 4$

$\Rightarrow \left|\frac{-24+14k-42}{k+1}\right| = 4 \Rightarrow \left|\frac{2(7k-33)}{k+1}\right| = 4$

$\Rightarrow 7k-33 = \pm 2(k+1) \Rightarrow 5k = 35, 9k = 31$

$\Rightarrow k = 7, \frac{31}{9}$

35. Ans.3

sol: A,B,C are the vertices of a right angled triangle with AC as its hypotenuse

$\Rightarrow AB^2 + BC^2 = AC^2 \Rightarrow (h-1)^2 + (k-1)^2 + (1-2)^2 + (1-1)^2$

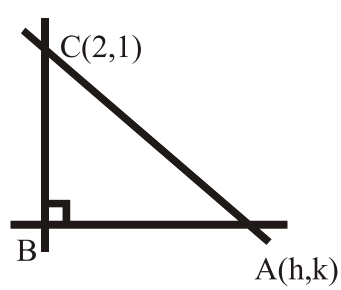
$= (h-2)^2 + (k-1)^2$

$\Rightarrow h^2 - 2h + 1 + 1 = h^2 - 4h + 4$

$\Rightarrow 2h = 2 \Rightarrow h = 1$. Given area of $\Delta = 1$

$\Rightarrow 1 = \frac{1}{2}\sqrt{(h-1)^2 + (k-1)^2} \Rightarrow 2 = \sqrt{0 + (k-1)^2}$

$\Rightarrow (k-1)^2 = 4 \Rightarrow k-1 = \pm 2 \Rightarrow k = 1 \pm 2 \Rightarrow k = -1, 3$



36. Ans 3

Sol: A(k,2–2k), B(–k+1,2k), C(–4–k,6–2k)

slope of AB = Slope of

$$AC \Rightarrow \frac{2k-2+2k}{-k+1-k} = \frac{6-2k-2+2k}{-4-k-k}$$

$$\Rightarrow \frac{4k-2}{1-2k} = \frac{4}{-2(k+2)} \Rightarrow 2(2k-1)(k+2)$$

$$= -2(1-2k) \Rightarrow (2k-1)(k+2) = -(1-2k)$$

$$= -(1-2k) \Rightarrow (2k-1)(k+2-1) = 0$$

$$\Rightarrow k = 1/2, -1$$

37. Ans.2

Sol: Let ‘a’ be the side of the square PQRS.

$\therefore$ 4a= 12 $\Rightarrow$ PQ = 3, QT = 4, PT = 5

$$\frac{\text{Area of the square PQRS}}{\text{Area of } \Delta \text{PQT}}$$

$$= \frac{a^2}{\frac{1}{2}PQ \times QT} = \frac{2a^2}{PQ.QT} = \frac{2(3)^2}{3\times 4} = \frac{3}{2}$$

Area of the square PQRS = $\frac{3}{2}$(Area of Δ PQT)

38. Ans.4

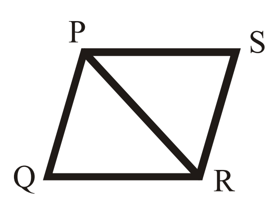
Sol: Let a,b (a>b) be the adjacent sides of a rectangle

$\therefore$ Length of the diagonal = $\sqrt{a^2+b^2}$

$$\sqrt{a^2+b^2} = \frac{a}{2} + b \Rightarrow a^2 + b^2 = \left(\frac{a+2b}{2}\right)^2$$

$$\Rightarrow \Rightarrow 4(a^2+b^2) = a^2 + 4b^2 + 4ab \Rightarrow 3a^2 = 4ab \Rightarrow 3a = 4b \Rightarrow b/a = 3/4$$

$$\Rightarrow b : a = 3 : 4$$



39. Ans.2

Sol: $a = 3, b = 4, c = 5 \therefore \angle C = 90°$, Δ = Area of the triangle $= \frac{1}{2} \times 3 \times 4 = 6$, $s = \frac{a+b+c}{2} = 6$

In radian (r) = $= \frac{\Delta}{s} = \frac{6}{6} = 1, \frac{c}{\sin C} = 2R \Rightarrow c = 2R \sin C \Rightarrow 5 = 2R \sin 90° \Rightarrow R = \frac{5}{2}$

$$\frac{(\text{Area of } S_1)}{(\text{Area of } S_2)} = \frac{\pi r^2}{\pi R^2} = \frac{1^2}{(5/2)^2} = \frac{4}{25}$$

40. Ans.3

Sol: P=Area of the first triangle $= \frac{1}{2}\begin{vmatrix} b-c & b-a \\ c-a & c-b \end{vmatrix}$

Area of the second triangle $= \frac{1}{2}\begin{vmatrix} ac-b^2-ab+c^2 & ab-c^2-bc+a^2 \\ ac-b^2-bc+a^2 & ab-c^2-ac+b^2 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} c^2-b^2+a(c-b) & a^2-c^2+b(a-c) \\ a^2-b^2+c(a-b) & b^2-c^2+a(b-c) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} (a+b+c)(c-b) & (a+b+c)(a-c) \\ (a+b+c)(a-b) & (a+b+c)(b-c) \end{vmatrix}$$

$$= \frac{1}{2}(a+b+c)^2\begin{vmatrix} c-b & a-c \\ a-b & b-c \end{vmatrix} = p(a+b+c)^2.$$

41. Ans.1

Sol: If A = (x,y) then $[x-(a+b)]^2+[y-(b-a]^2$

$= [x-(a-b)]^2+[y-(a+b)]^2$.

$\Rightarrow [(x-a)-b]^2 - [(x-a)+b]^2 = [(y-b)-a]^2 - [(y-b)+a]^2 \Rightarrow 4b(x-a) = 4(y-b)a$

$\Rightarrow bx = ay$ which is satisfied by (1).

42. Ans.1

Sol: Area of the triangle $= \frac{1}{2}\left|\begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix}\right| = \frac{1}{2}|ab(r-1)(s-r)| = \frac{1}{2}$

$|ab(r-1)(r+1)2| = |ab(r^2-1)|$

43. Ans.3

Sol: Given that, the triangle ABC is isosceles

$\therefore |AB| = |AC|$

Let the coordinate of A be A(h,k)

$\therefore \sqrt{(h-1)^2+(k-3)^2} = \sqrt{(h+2)^2+(k-7)^2} \Rightarrow (h-1)^2+(k-3)^2 = (h+2)^2+(k-7)^2$

$\Rightarrow -2h+1-6k+9 = 4h+4-14k+49$

$\Rightarrow 6h-8k+43 = 0 \rightarrow (1)$

Since, the area of triangle is 10 sq. unit (given)

Area $(\Delta ABC) = \frac{1}{2}|BC||AD| \Rightarrow \left|\frac{1}{2}(5)\sqrt{\left(h+\frac{1}{2}\right)+(k-5)^2}\right| = \frac{25}{6}$

$\Rightarrow \left|\sqrt{\left(h+\frac{1}{2}\right)+(k-5)^2}\right| = \frac{5}{3}$

$\Rightarrow \left(h+\frac{1}{2}\right)^2+(k-5)^2 = \frac{25}{9} \Rightarrow \left(\frac{8k-43}{6}+\frac{1}{2}\right)^2+(k-5)^2 = \frac{25}{9}$

$\Rightarrow (4k-20)^2+9(k-5)^2 = 25 \Rightarrow 25(k-5)^2 = 25$

$\Rightarrow |k-5| = 1 \Rightarrow k-5 \pm 1$

$\Rightarrow k = 6$ or 4 and hence h = 5/6 or –11/6.

Therefore, the vertex A of the isosceles Δ ABC is A(5/6,6) or A(–11/6,4)

44. Ans.2

Sol: Let the point P be (h,k) the PA = PB $\Rightarrow PA^2 = PB^2 \Rightarrow (h-3)^2+(k-4)^2 =$

$(h-5)^2+(k+2)^2 \Rightarrow h-3k-1=0 \Rightarrow h = 3k+1 \rightarrow (1).$

Area of Δ PAB = 10 $\Rightarrow \frac{1}{2}\begin{vmatrix} h & k & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 10 \Rightarrow \frac{1}{2}\begin{vmatrix} h & k & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 20$

$\Rightarrow |h(4+2)-k(3-5)+1(-6-20)| = 20 \quad \Rightarrow |6h+2k-26| = 20 \Rightarrow 3h+k-13 = \pm 10$

$\Rightarrow 3(3k+1)+k-13 = \pm 10 \Rightarrow 9k-10 = \pm 10 \Rightarrow k = 2 \text{ or } 0.$

$\therefore h = 7$ or 1, $\therefore$ P=(7,2) or (1,0).