STRAIGHT LINE

SYNOPSIS AND FORMULAE

1. Inclination of a Line

i. The angle between the line and the positive direction of x-axis is called the inclination of the line.

ii. If 'θ' is the inclination of a ling the $\theta \pm \frac{\pi}{2}$ is the inclination of its perpendicular line according at θ is acute or obtuse.

iii. Inclination of parallel lines are equal.

2. Slope of Line

i. If 'θ' is the inclination of a line, then 'tan θ' is called its 'slope' and is denoted by 'm'.

ii. The slope of a horizontal line (i.e. a line parallel to x-axis) is zero.

iii. The slope of a vertical line (i.e. a line perpendicular to x-axis) is not defined.

iv. The slopes of parallel line are equal.

v. If m_1, m_2 are the slopes of two mutually perpendicular lines, then $m_1m_2 = -1$ (non vertical).

vi. The slope of a line passing through $A(x_1, y_1)$, $B(x_2, y_2)$ is $\frac{y_1 - y_2}{x_1 - x_2}$, when $x_1 \neq x_2$.

vii. If a line cuts x-axis at A (a, 0) and y-axis at B (0, b),then the slope of AB is $\frac{-b}{a}$, where 'a' is the x-intercept and 'b' is the y-intercept.

3. Different forms of a Straight Line

i. **Slope-intercept Form:** equation of the line having slope 'm' and y-intercept 'c is

$y = mx + c$.

ii. Point-Slope Form : equation of the line having slope 'm' and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

iii. **Two-point Form:** Equation of the line passing through points (x_1, y_1) and $(x_2\ y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

iv. **Intercept Form:** Equation of the line which has x-intercept 'a and y-intercept 'b' is

$$\frac{x}{a} + \frac{y}{b} = 1.$$

v. **Normal or perpendicular form:** Equation of the line having 'p' as its perpendicular distance from the origin and 'α' as the inclination of its normal from the origin, is $x \cos\alpha + y \sin\alpha = p$, $(0 \le \alpha < 360)$.

vi**. Symmetric Form:** Equation of the line passing through (x_1, y_1) and having inclination 'θ' with the x-axis is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r$ where $\theta \pm \frac{n\Pi}{2}$ and 'r' is the distance of (x_1, y_1) from (x, y) along the lien. The parametric equations of symmetric form are $x = x_1 + r\cos\theta$, $y = y_1 + r\sin\theta$, where $|r|$ is the distance between (x, y) and (x_1, y_1).

4. General form of Straight Line:

Every first degree equation in x and y is $ax + by + c = 0$, $(a, b) \ne (0, 0)$ represents a straight line.

i. Slope of the line = $\frac{-a}{b}$

ii. X-intercept = $\frac{-c}{a}$.

iii. y-intercept = $\frac{-c}{b}$.

iv. Area of the triangle formed by this line and the coordinate axes = $\frac{c^2}{2|ab|}$.

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v. Length of the perpendicular from the origin is $\frac{|c|}{\sqrt{a^2+b^2}}$

vi. Length of the perpendicular from (x_1, y_1) is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

vii. The distance between the parallel lines ax + by + c_1 = 0, ax + by + c_1 = 0 is $\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$.

5. Equation of the line passing though (x_1, y_1):

i. And parallel to ax + by + c = 0 is a(x – x_1) + b(y – y_1) = 0.

ii. And perpendicular to ax + by + c = 0 is b (x – x_1) – a(y – y_1) = 0.

6. If slopes of two lines are m_1, m_2 and θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$.

7. Equation of any line passing through (x_1, y_1) and making an angle α with the line ax + by + c = 0 is $y - y_1 = \tan(\theta \pm \alpha)(x - x_1)$, where $\tan\theta = \frac{-b}{a}$.

8. Let the equations of two lines be $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$**.**

i. any line passing through the intersection of $L_1 = 0$ and $L_2 = 0$ is $L_1 + \lambda L_2 = 0$.

ii. The point of intersection of $L_1 = 0$, $L_2 = 0$ is $\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}.\frac{c_1a_2-c_2a_1}{a_{1b_2-a_2b_1}}\right)$.

iii. If θ is the acute angle between $L_1 = 0$, $L_2 = 0$ then $\cos\theta = \frac{|a_1a_2+b_1b_2|}{\sqrt{a_1{}^2+b_1{}^2}\sqrt{a_2{}^2+b_2{}^2}}$, $\tan\theta = \left|\frac{a_1b_2-a_2b_1}{a_1a_2+b_1b_2}\right|$ and $\sin\theta = \frac{|a_1b_2-a_2b_1|}{\sqrt{a_1{}^2+b_1{}^2}\sqrt{a_2{}^2+b_2{}^2}}$.

iv. Condition for $L_1 = 0$, $L_2 = 0$ to be parallel is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

v. Condition for $L_1 = 0$, $L_2 = 0$ to be perpendicular is $a_1a_2 + b_1b_2 = 0$.

vi. Condition for $L_1 = 0$, $L_2 = 0$ to be coincident is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

9. The ration in which $L = ax + by + c = 0$ divides the line segment joining (x_1, y_1) and (x_2, y_2) is $\frac{-L_{11}}{L_{22}}$ **Where** $L_{11} = ax_1 + by_1 + c$; $L_{22} = ax_2 + by_2 + c$.

10. The points (x_1, y_1) and (x_2, y_2) are on the same side or on the opposite sides of $L = 0$ according as the signs of L_{11}, L_{22} are the same or different.

11. Equation of the angular bisectors of the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = \mathbf{0}$ **are** $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$.

12. i. If c_1, c_2 are positive, then $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ is the bisector of the angle containing the origin.

ii. If c_1, c_2 are positive, then $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = -\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the bisector of the angle not containing the origin.

iii. In the case of the bisector of the angle containing the origin, the angle between $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_1 = 0$, is acute or obtuse according as $a_1a_2 + b_1b_2 < 0$ or > 0.

13. Area of the triangle formed by $y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$ is $\frac{1}{2}\left|\sum\frac{|c_1-c_2|^2}{m_1-m_2}\right|$.

14. Area of the parallelogram ABCD is $\frac{P_1P_2}{\sin\theta}$ where P_1, P_2 are the distance between the parallel sides and θ is the angle between the adjacent sides of the parallelogram.

15. Area of the parallelogram flowed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(c_1-d_1)(c_2-d_2)}{a_1b_2-a_2b_1}\right|$.

16. The foot of the perpendicular from (x_1, y_1) on to the line $ax + by + c = 0$ is $(x_1 + a\lambda, y_1 + b\lambda)$, where $\lambda = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$

17. The image of the point (x_1, y_1) w.r.t. the line $ax + by + c = 0$ is $(x_1 + a\lambda, y_1 + b\lambda)$, where $\lambda = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$.

18. The distance of the point $P(x_1, y_1)$ from the line $ax + by + c = 0$ in the direction making an angle θ with x-axis is $\left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$.

19. Let (x_1, y_1) divide the segment AB between the coordinate axes in the ratio $\lambda : \mu$.

The equation of AB is $\frac{\lambda x}{x_1} + \frac{\mu y}{y_1} = \lambda + \mu$.

20. In-centre of the triangle formed by $lx + my + n = 0$ and two coordinate axis is

$\left(\frac{a|b|}{|a|+|b|+\sqrt{a^2+b^2}}, \frac{b|a|}{|a|+|b|b|+\sqrt{a^2+b^2}}\right)$ Where ‘a’ and ‘b’ are the intercepts made by the line on the axis.