Digital Electronics

Boolean Algebra & Logic Gates

Boolean Expressions

A Boolean expression or a function is an expression which consists of binary variables joined by the Boolean connectives AND and OR along with NOT operation.

Any Boolean expression can be expressed in two forms:

a) Canonical form

b) Standard form

Canonical Form:

An expanded form of Boolean expression, where each term contains all Boolean variables in their true or complemented form, is known as the canonical form of the expression.

a) **Sum of minterms:** Any Boolean function can be expressed as a sum of minterms expression. A minterm is a standard product which consists of all variables in either complemented or un-complemented form for which the output is 1. For example,

$$Y = A'BC + AB'C' + ABC$$
$$= \sum m(3,4,7)$$

is a sum of minterms expression with three variables.

b) **Product of maxterms:** Any Boolean function can be expressed as a product of maxterms expression. A maxterm is a standard sum which consists of all variables in either complemented or un-complemented form for which the output is 0. For example,

$$Y = (A' + B' + C)(A + B + C')(A + B + C) = \prod M\,(0,1,6)$$

is a product of maxterms expression with three variables.

Standard Form:

A simplified form of a Boolean expression which may consist of one or more number of variables in each term in either complemented or un-complemented form is known as Standard form of the expression.

a) **Sum of Products (SOP):** The sum of products is a Boolean expression containing AND terms, called Product terms, of one or more literals each; the sum denotes the ORing of these terms. For example,

$$Y = A'B + BC' + AC$$

is a SOP expression with three variables.

b) **Product of Sums (POS):** It is a Boolean expression containing OR terms called Sum terms and the product denotes the ANDing of these terms.

$$Y = A(A + B')(B + C')$$

is a POS expression with three variables.

**** Canonical form is obtained when a function is taken from a truth table. When implementing a Boolean function with gates, standard form is preferred.**

Simplification of Boolean expressions:

The primary objective of all simplification procedures is to obtain an expression that has the minimum number of terms. Obtaining an expression with the minimum number of literals is usually the secondary objective. The Boolean functions can be simplified by using

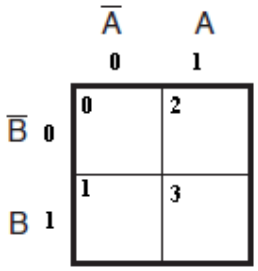
a) Boolean Laws and theorems
b) K maps
c) Quine Mc-Cluskey or Tabulation Method

Simplification using K-maps:

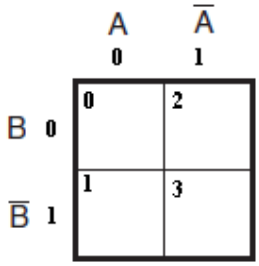
A Karnaugh map is a graphical representation of the logic system. It can be drawn directly from either minterm (sum-of-products) or maxterm (product-of-sums) Boolean expressions. Drawing a Karnaugh map from the truth table involves an additional step of writing the minterm or maxterm expression depending upon whether it is desired to have a minimized sum-of-products or a minimized product of sums expression.

An n-variable Karnaugh map has 2^n squares, and each possible input is allotted a square. In the case of a minterm Karnaugh map, '1' is placed in all those squares for which the output is '1', and '0' is placed in all those squares for which the output is '0'. 0s are omitted for simplicity. An 'X' is placed in squares corresponding to 'don't care' conditions.

a) Two Variable K-map:

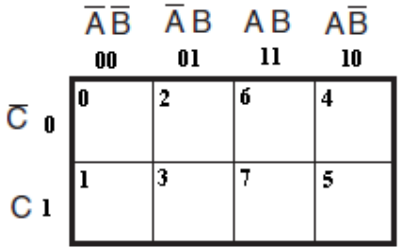


Sum of minterms Representation

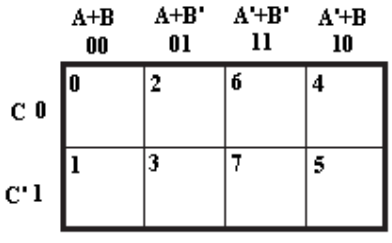


Product of maxterms representation

b) Three Variable K-map:

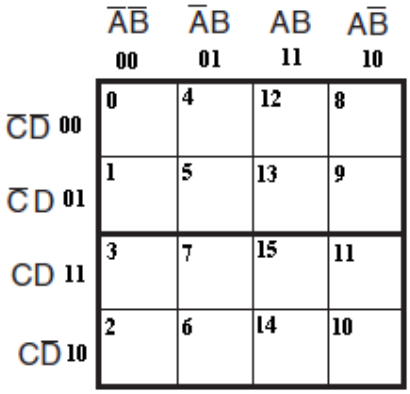


Sum of minterms Representation

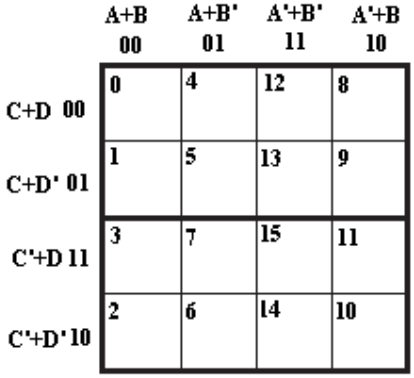


Product of maxterms representation

c) Four Variable K-map:



$\bar{C}\bar{D}$ 00	0	4	12	8
$\bar{C}D$ 01	1	5	13	9
CD 11	3	7	15	11
$C\bar{D}$ 10	2	6	14	10

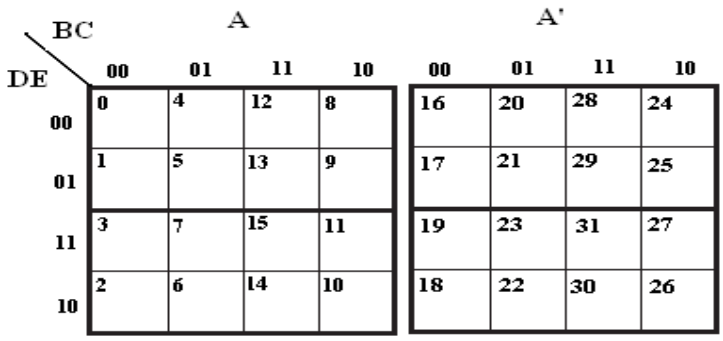


C+D 00	0	4	12	8
C+D' 01	1	5	13	9
C'+D 11	3	7	15	11
C'+D' 10	2	6	14	10

Sum of minterms Representation

Product of maxterms representation

d) Five Variable K-map:



BC \ DE	A 00	A 01	A 11	A 10	A' 00	A' 01	A' 11	A' 10
00	0	4	12	8	16	20	28	24
01	1	5	13	9	17	21	29	25
11	3	7	15	11	19	23	31	27
10	2	6	14	10	18	22	30	26

Sum of minterms Representation

Product of maxterms representation