9. RANDOM VARIABLE AND DISTRIBUTIONS

PREVIOUS EAMCET BITS

1. If m and σ^2 are the mean and variance of the random variable X, whose distribution is given by

X = x :	0	1	2	3
P(X = x)	1/3	1/2	0	1/6

Then **[EAMCET 2009]**

1) $m = \sigma^2 = 2$ 2) $m = 1, \sigma^2 = 2$ 3) $m = \sigma^2 = 1$ 4) $m = 2, \sigma^2 = 1$

Ans:

Sol : $m = 0 + \frac{1}{2} + 0 + \frac{3}{6} = 1$

$\sigma^2 = \sum_n x_n^2.p_n - m^2 = 1$

2. If X is a binomial variate with the range $\{0,1,2,3,4,5,6\}$ and P(X =2) = 4 P(X = 4), then the parameter p of X is **[EAMCET 2009]**

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) $\frac{2}{3}$ 4) $\frac{3}{4}$

Ans: 1

Sol: Given $n = 6, P(X = 2) = 4P(X = 4)$

$\Rightarrow 6_{c_2} p^2 q^4 = 4 \times 6_{c_4} p^4 q^2$

$\Rightarrow q = 2p \Rightarrow 1 - p = 2p$

$\Rightarrow p = 1/3$

3. The distribution of a random variable X is given below

X = x	– 2	– 1	0	1	2	3
P(X = x)	$\frac{1}{10}$	K	$\frac{1}{5}$	2k	$\frac{3}{10}$	k

the value of k is **[EAMCET 2008]**

1) $\frac{3}{10}$ 2) $\frac{2}{10}$ 3) $\frac{3}{10}$ 4) $\frac{7}{10}$

Ans:

Sol: $\sum P(X = x) = 1 \Rightarrow \frac{1}{10} + k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$

$\Rightarrow 4k = \frac{4}{10} \Rightarrow k = \frac{1}{10}$

4. If X is a Poisson variate such that $P(X=1)P(X=2)$, then $P(X=4)$ **[EAMCET 2008]**

1) $\frac{1}{2e^2}$ 2) $\frac{1}{3e^2}$ 3) $\frac{2}{3e^2}$ 4) $\frac{1}{e^2}$

Ans: 3

Sol: $P(X=1)=P(X=2)\Rightarrow \frac{e^{-\lambda}\lambda}{1!}=\frac{e^{-\lambda}\lambda^2}{2!}$

$\Rightarrow \lambda=2.P(X=4)=\frac{e^{-\lambda}\lambda^4}{4!}=\frac{e^{-2}2^4}{4!}=\frac{2}{3e^2}$

5. The mean and standard deviation of a binomial variate X are 4 and $\sqrt{3}$ respectively. Then $P(X\geq 1)$ is equal to **[EAMCET 2007]**

1) $1-\left(\frac{1}{4}\right)^{16}$ 2) $1-\left(\frac{3}{4}\right)^{16}$ 3) $1-\left(\frac{2}{3}\right)^{16}$ 4) $1-\left(\frac{1}{3}\right)^{16}$

Ans: 2

Sol: Mean = np = 4, variance nqp = 3

On solving, we get $q=\frac{3}{4}, n=16, p=\frac{1}{4}$

Now $p(X\geq 1)=1-p(X=0)=1-{}^{n}C_0p^0q^{n-0} = 1-\left(\frac{3}{4}\right)^{16}$

6. The probability distribution of a random variable X is given by

X = x	0	1	2	3	4
P(X = x)	0.4	0.3	0.1	0.1	0.1

The variance of X is **[EAMCET 2007]**

1) 1.76 2) 2.45 3) 3.2 4) 4.8

Ans: 1

Sol: Given

X = x	P(X = x)	xP(X = x)	x^2P
0	0.4	0	0
1	0.3	0.3	0.3
2	0.1	0.2	0.4
3	0.1	0.3	0.9
4	0.1	0.4	1.6

Mean $\overline{x}=\sum_{i=0}^{4}p_ix_i=1.2$

Variance $=\sum_{i=0}^{4} p_i x_i^2 - \overline{x}^2 = 3.20 - 1.44 = 1.76$

7. In a book of 500 pages, it is found that there are 250 typing errors. Assume that Poisson law holds for the number of errors per page. Then, the probability that a random sample of 2 pages will contain no error, is **[EAMCET 2006]**

1) $e^{-0.3}$ 2) $e^{-0.5}$ 3) e^{-1} 4) e^{-2}

Ans: 3

Sol: Here number of errors per page = $\frac{250}{500} = \frac{1}{2}$

and n = 2

$\therefore\ \lambda = np = 2 \times \frac{1}{2} = 1$ and probability of no error $P(X=0) = \frac{e^{-1} \times (1)^0}{0!} = e^{-1}$

8. If the range of a random variable X is {0, 1, 2, 3, 4, …..} with $P(X=k) = \frac{(k+1)a}{3^k}$ for $k \geq 0$, then a is equal to **[EAMCET 2005]**

1) $\frac{2}{3}$ 2) $\frac{4}{9}$ 3) $\frac{8}{27}$ 4) $\frac{16}{81}$

Ans: 2

Sol: Given that $P(X=k) = \frac{(k+1)}{3^k}$ for $x \in \{0,1,2,....\infty\}$

As we know that $P(0)+P(1)+P(2)+.......\infty = 1$

$\Rightarrow a + \frac{2a}{3} + \frac{3a}{3^2} + ...\infty = 1$ …..(i)

$$S = a\left(1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+....\infty\right) \Rightarrow \begin{array}{l} \frac{1}{3}S = a\left(\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+....\infty\right) \\ \hline S-\frac{1}{3}S = a\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+...\infty\right) \end{array}$$

$$\Rightarrow \frac{2}{3}S = a\left(\frac{1}{1-\frac{1}{3}}\right) \Rightarrow \frac{2}{3}S = \frac{3a}{2} \Rightarrow S = \frac{9a}{4}$$

from equation (i) $\Rightarrow \frac{9a}{4} = 1 \Rightarrow a = \frac{4}{9}$

9 For a binomial variate X with n = 6, if P(X = 2) = 9P(X = 4), then its variance is **[EAMCET 2005]**

1) $\frac{8}{9}$ 2) $\frac{1}{4}$ 3) $\frac{9}{8}$ 4) 4

Ans: 3

Sol: Given that n = 6 and $P(X=2)=9P(X=4)$

$\Rightarrow {}^6C_2p^2q^4 = 9.{}^6C_4p^4q^2$

$\Rightarrow 9p^2 = q^2 \Rightarrow p = \frac{1}{3}q$

$\because$ we know that p + q = 1

$\Rightarrow \frac{q}{3}+q=1 \Rightarrow q=\frac{3}{4}$ and $p=\frac{1}{4}$

$\therefore$ variance = npq = $6.\frac{1}{4}.\frac{3}{4}=\frac{9}{8}$

10. A person who tosses an unbiased coin gains two points for turning up a head and loses one point for a tail. If three coins are tossed and the total score X is observed, then the range of x is **[EAMCET 2004]**

1) $\{0.3.6\}$ 2) $\{-3,0,3\}$ 3) $\{-3,0,3,6\}$ 4) $\{-3,3,6\}$

Ans: 3

Sol: Since it is given that for tossing a coin, if head will come down it will give two point and for tail comes down it loose one point.

There are four case arise : Case(i) If all three tails comes out, then his points = – 1 –1 –1 = -3

Case (ii) If two tails and one head comes out, then his points = – 1 –1 + 2 = 0

Case (iii) If one tail and two heads comes out, then this points = –1 + 2 + 2 =3

Case (iv) If all three heads comes out, then his points = 2 + 2 + 2 = 6

$\therefore$ Range = {– 3, 0, 3, 6}

11. If X is a Poisson variate with P(X = 0) = 0.8, then the variance of X is **[EAMCET 2004]**

1) $\log_e 20$ 2) $\log_{10} 20$ 3) $\log_e 5/4$ 4) 0

Ans: 3

Sol: Poisson distribution $P(X)=\frac{e^{-m}m^x}{x!}$

$\therefore$ $P(X=0)=\frac{e^{-m}1}{1} \Rightarrow 0.8=e^{-m} \Rightarrow -m=\log_e 0.8$

$\Rightarrow m=\log_e\frac{10}{8}=\log_e\frac{5}{4}$

As we know in a Poisson distribution variance = m

Variance = $\log_e\frac{5}{4}$

12. For a Poisson variate X, if P(X = 2) = 3P(X = 3), then the mean of X is **[EAMCET 2003]**

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

Ans: 1

Sol: We known that $P(X=k)=\frac{\lambda^k e^{\lambda}}{k!}$

Since, $P(X=2)=3P(X=3)$

$\Rightarrow \frac{\lambda^2 e^{\lambda}}{2!}=3.\frac{\lambda^3 e^{\lambda}}{3!}\Rightarrow \lambda=1$

$\therefore$ Mean of Poisson distribution = $\lambda \Rightarrow 1$

13. A random variable X takes the values 0, 1 , 2, 3 and its mean is 13. If P(X = 3) = 2P(X = 1) an P(X = 2) = 0.3, then P(X = 0) is equal to **[EAMCET 2003]**

1) 0.1 2) 0.2 3) 0.3 4) 0.4

Ans: 4

Sol: Given that Mean $=\sum X_k P(X=k)=1.3$

$X_0P(X=0)+X_1P(X=1)+X_2P(X=2)+X_3P(X=3)=1.3$

$\Rightarrow 0.P(X=0)+1.P(X=1)+2.P(X=2)+3.P(X=3)=1.3$

$\Rightarrow P(X=1)+2(0.3)+3.2P(X=1)=1.3$

$\Rightarrow 7P(X=1)=0.7\Rightarrow P(X=1)=0.1$

Now, $P(X=3)2P(X=1)=2(0.1)=0.2$

Also, $P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$

$\Rightarrow P(X=0)+0.1+0.3+0.2=1\Rightarrow P(X=0)=1-0.6=0.4$

14. In a binomial distribution the probability of getting success is 1/4 and the standard deviation is 3. Then its mean is **[EAMCET 2002]**

1) 6 2) 8 3) 10 4) 12

Ans:4

Sol: Given that $P=\frac{1}{4}$ and $q=1-\frac{1}{4}=\frac{3}{4}$

S.D = 3 $\Rightarrow \sqrt{npq}=3\Rightarrow npq=9\Rightarrow n.\frac{1}{4}.\frac{3}{4}=9$ = n = 48

Mean = np = $48\times\frac{1}{4}=12$

15. If the mean of a Poisson distribution is 1/2 then the ratio of P(X = 3) to P(X = 2) is **[EAMCET 2002]**

1) 1 : 2 2) 1 : 4 3) 1 : 6 4) 1 : 8

Ans: 3

Sol: Given that $\lambda = \frac{1}{2}$, Now $P(X = n) = \frac{\lambda^n}{n!}e^{\lambda}$

$$\therefore\ P(X=3) = \frac{\left(\frac{1}{2}\right)^3}{3!}e^{1/2} \text{ and } P(X=2) = \frac{\left(\frac{1}{2}\right)^2}{2!}e^{1/2}$$

$$\frac{P(X=3)}{P(X=2)} = \frac{\frac{\left(\frac{1}{2}\right)^3 e^{1/2}}{3!}}{\frac{\left(\frac{1}{2}\right)^2 e^{1/2}}{2!}} = \frac{\left(\frac{1}{2}\right)^3 e^{1/2}\,2!}{\left(\frac{1}{2}\right)^2 e^{1/2}\,3!} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

16. A random variable X takes the values 0, 1 and 2. If P(X =1) = P(X = 2) and P(X = 0) = 0.4, then the mean of the random variable X is **[EAMCET 2002]**

1) 0.2 2) 0.7 3) 0.5 4) 0.9

Ans: 4

Sol: We have P(X =1) = P(X =2) ………..(i)

$$\frac{\lambda^1}{1!}e^{\lambda} = \frac{\lambda^2}{2!}e^{\lambda} \Rightarrow \lambda = 2$$

also P(X = 0) + P(X = 1) + P(X = 2) = 1

$$\Rightarrow 0.4 + P(X=1) + P(X=2) = 1$$

$$\Rightarrow P(X=1) + P(X=2) = 0.6 = \frac{6}{10} = \frac{3}{5}$$

Also, $P(X=1) + P(X=1) = \frac{3}{5}$ [from (i)]

$$\Rightarrow P(X=1) = \frac{3}{10} \Rightarrow P(X=1) = P(X=2) = \frac{3}{10}$$

Mean $X_0 P(X=0) + X_1 P(X=1) + X_2 P(X=2)$

$$= 0 + 1.\frac{3}{10} + 2.\frac{3}{10} = \frac{9}{10} = 0.9$$

17. Find the binomial probability distribution whose mean is 3 and variance is **[EAMCET 2001]**

1) $\left(\frac{2}{3}+\frac{1}{3}\right)^9$ 2) $\left(\frac{5}{3}+\frac{2}{3}\right)^9$ 3) $\left(\frac{3}{3}+\frac{1}{2}\right)^9$ 4) None of these

Ans: 1

Sol: We have $np = 3, npq = 2 \Rightarrow q = \frac{2}{3}$

$\therefore \; p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}, n = 9$

Hence, the binomial distribution is $(q+p)^n = \left(\frac{2}{3} + \frac{1}{3}\right)^9$

18. For a binomial variate X, if n = 4 and $P(X=4) = 6P(X=2)$, then the value of p is

[EAMCET 2001]

1) $\frac{3}{7}$ 2) $\frac{4}{7}$ 3) $\frac{6}{7}$ 4) $\frac{5}{7}$

Ans: 3

Sol: We have, $P(x=4) = 6P(x=2)$

$\Rightarrow {}^4C_4 p^4 q^0 = 6.{}^4C_2 p^2 q^2$

$\Rightarrow p^2 = 36q^2$

$\Rightarrow p = 6q = 6(1-p) \Rightarrow p = \frac{6}{7}$

19. For all values of a and b the line $(a+2b)x + (a-b)y + (a+5b) = 0$ passes through the point

[EAMCET 2001]

1) $(-1,2)$ 2) $(2,-1)$ 3) $(-2,1)$ 4) $(1,-2)$

Ans: 3

Sol: Let the line passes through a point whose co-ordinates are (–2 , 1) then

$(a+2b)(-2) + (a-b)(1) + a + 5b$

$= -2a - 4b + a - b + a + 5b$

$= -2a - 5b + 2a + 5b = 0$

20. The probability distribution of a random variable X is given below, then k is equal to

N	1	2	3	4
P(X = n)	2k	4k	3k	K

[EAMCET 2000]

1) 0.1 2) 0.2 3) 0.3 4) 0.4

Ans: 1

Sol: We have, $P(X=1) + P(x=2) + P(X=3) + P(X=4) = 1$

$$\Rightarrow 2k + 4k + 3k + k = 1 \Rightarrow k = \frac{1}{10} = 0.1$$