QUADRATIC EXPRESSIONS

PREVIOUS EAMCET BITS

1. The roots of $(\mathbf{x-a})(\mathbf{x-a-1})+(\mathbf{x-a-1})(\mathbf{x-a-2})+(\mathbf{x-a})(\mathbf{x-a-2})=\mathbf{0}$, $a \in IR$ always :

   1) equal 2) imaginary 3) real and distinct 4) rational and equal

   Ans: 3 [EAMCET 2009]

Sol : Put a = 0

$x(x-1)+(x-1)(x-2)+(x)(x-2)=0$

$3x^2-6x+2=0 \Rightarrow 3x^2-6x+2=0$

$\mathbf{\Delta=(6)^2-4(3)(2)}$

$\mathbf{\Delta} \neq 0$

$\therefore$ Roots are real and distinct.

2. Let $\mathbf{f(x)=x^2+ax+b}$, where $a,b \in IR$. If $\mathbf{f(x)=0}$ has all its roots imaginary, then the roots of $\mathbf{f(x)+f^1(x)+f^{11}(x)=0}$ are : [EAMCET 2009]

   1) real and distinct 2) imaginary 3) equal 4) rational and equal

   Ans: 2

Sol. Given roots of $\mathbf{f(x)}$ are imaginary $\Rightarrow \Delta < 0$ $\mathbf{a^2-4b<0}$

$\mathbf{f^1(x)=2x+a}$

$\mathbf{f^{11}(x)=2}$

$\mathbf{f(x)+f^1(x)+f^{11}(x)=0}$

$\mathbf{x^2+ax+b+2x+a+2=0}$

$\Rightarrow \mathbf{x^2+(a+2)x+(a+b+2)=0}$

$\therefore \mathbf{\Delta=(a+2)^2-4(1)(a+b+2)}$

$=a^2+4a+4-4a-4b-8$

$\mathbf{=a^2-4b-4<0}$

$\therefore$ Roots are imaginary.

3. Let $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be the roots of the quadratic equations $\mathbf{ax^2+bx+c=0}$ observe the lists given below. [EAMCET 2008]

LIST - I	LIST – II
	A) $\mathbf{(ac^2)^{1/3}+(a^2c)^{1/3}+b=0}$
i) $\alpha=\beta \Rightarrow$	B) $\mathbf{2b^2=9ac}$
ii) $\boldsymbol{\alpha=2\beta} \Rightarrow$	C) $\mathbf{b^2=6ac}$
iii) $\boldsymbol{\alpha=3\beta} \Rightarrow$	D) $\mathbf{3b^2=16ac}$
iv) $\boldsymbol{\alpha=\beta^2} \Rightarrow$	E) $\mathbf{b^2=4ac}$
	F) $\mathbf{(ac^2)^{1/3}+(a^2c)^{1/3}=b}$

The correct match to List – I from List – Ii is

	i	ii	iii	iv		i	ii	iii	iv		i	ii	iii	iv		i	ii	iii	iv
1)	E	B	D	F	2)	E	B	A	D	3)	E	D	B	F	4)	E	B	D	A

Ans: 4

Sol : If the roots are in the ratio m : n then $\frac{b^2}{ac} = \frac{(m+n)^2}{mn}$

i) $\alpha = \beta \Rightarrow \frac{\alpha}{\beta} = \frac{1}{1} \Rightarrow \frac{b^2}{ac} = \frac{(1+1)^2}{1x1} \Rightarrow b^2 = 4ac$

ii) $\alpha = 2\beta \Rightarrow \frac{\alpha}{\beta} = \frac{2}{1} \Rightarrow \frac{b^2}{ac} = \frac{(2+1)^2}{2x1} \Rightarrow 2b^2 = 9ac$

iii) $\alpha = 3\beta \Rightarrow \frac{\alpha}{\beta} = \frac{3}{1} \Rightarrow \frac{b^2}{ac} = \frac{(3+1)^2}{3x1} \Rightarrow 3b^2 = 16ac$

iv) $\alpha = \beta^2 \Rightarrow \beta + \beta^2 = \text{-b/a}, \ \beta^3 = \frac{c}{a}$

$\Rightarrow \left(\frac{\mathbf{c}}{\mathbf{a}}\right)^{1/3} + \left(\frac{\mathbf{c}}{\mathbf{a}}\right)^{2/3} = \mathbf{-b / a}$

$\Rightarrow \left(\mathbf{a^2c}\right)^{1/3} + \left(\mathbf{ac^2}\right)^{1/3} = \mathbf{-b}$

4. If $\boldsymbol{\alpha + \beta = -2}$ and $\boldsymbol{\alpha^3 + \beta^3 = -56}$ then the quadratic equation whose roots are $\boldsymbol{\alpha}$ **and** $\boldsymbol{\beta}$ is

1) $\mathbf{x^2 + 2x - 16 = 0}$ 2) $\mathbf{x^2 + 2x - 15 = 0}$ 3) $\mathbf{x^2 + 2x - 12 = 0}$ 4) $\mathbf{x^2 + 2x - 8 = 0}$

[EAMCET 2008]

Ans: 4

Sol : $\boldsymbol{\alpha^3 + \beta^3 = -56}$

$\boldsymbol{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -56}$

$\boldsymbol{(-2)^3 + 3\alpha\beta(-2) = -56}$

$\boldsymbol{-8 + 6\alpha\beta = -56}$

$\boldsymbol{\alpha\beta = -8}$

$\therefore$ Required equation is

$\boldsymbol{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$

$\mathbf{x^2 + 2x - 8 = 0}$

5. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ then r = [EAMCET 2007]

1) $a + 2b$ 2) $a + b + c$ 3) $ab + bc + ca$ 4) abc

Ans: 2

Sol. The equation heaving roots $\frac{\mathbf{1}}{\boldsymbol{\alpha}}, \frac{\mathbf{1}}{\boldsymbol{\beta}}$ is $\mathbf{cx^2 + bx + a = 0}$.

The equation having roots $\frac{1}{\alpha}-1, \frac{1}{\alpha}-1$ is

$\mathbf{c(x+1)^2+b(x+1)+a=0}$

$\Rightarrow \mathbf{cx^2+(2c+b)x+(c+b+a)=0}$

$\mathbf{px^2+qx+r=0}$

$\therefore$ r = a + b + c

6. The set of values of x for which the inequalities $x^2-3x-10<0, 10x-x^2-16>0$ hold simultaneously is [EAMCET 2007)

1) $(-2,5)$ 2) $(2,8)$ 3) $(-2,8)$ 4) $(2,5)$

Ans: 4

Sol. $\mathbf{x^2-3x-10<0 \Rightarrow (x-5)(x+2)>0}$

$\Rightarrow \mathbf{-2<x<5(or)x>3}$

$\mathbf{10x-x^2-16>0 \Rightarrow x^2-10x+16<0}$

$\Rightarrow \mathbf{(x-2)(x-8)<0}$

$\Rightarrow \mathbf{2<x<8}$

$\therefore$ common set = (2,5)

7. If $\sqrt{\mathbf{9x^2+6x+1}} < \mathbf{(2-x)}$ then [EAMCET 2006]

1) $\mathbf{x \in \left(-\frac{3}{2},\frac{1}{4}\right)}$ 2) $\mathbf{x \in \left(\frac{-3}{2},\frac{1}{4}\right]}$ 3) $\mathbf{x \in \left[\frac{-3}{2},\frac{1}{4}\right)}$ 4) x < 1/4

Ans: 1

Sol : $\sqrt{\mathbf{9x^2+6x+1}} < \mathbf{(2-x)}$

$\Rightarrow \sqrt{\mathbf{(3x+1)^2}} < \mathbf{(2-x)}$

Taking +ve sign

$\mathbf{3x+1<(2-x)}$

$\Rightarrow \mathbf{x<1/4}$

Taking –ve sign

$\mathbf{-3x-1<2-x}$

$\mathbf{x>\frac{-3}{2}}$

$x \in \left(\frac{-3}{2},\frac{1}{4}\right)$

8. If x is real, then the minimum value of $\frac{x^2-x+1}{x^2+x+1}$ is [EAMCET 2005]

1) 1/3 2) 3 3) 1/2 4)1

Ans: 1

Sol. Let $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow (y-1)x^2 + (y+1)x + (y-1) = 1$

x is real $\Rightarrow (y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow \frac{1}{3} \leq y \leq 3$

$\therefore$ Minimum value = 1/3

9. E_1: a + b + c = 0 if 1 is a root of $ax^2 + bx + c = 0$

E_2: b^2-a^2=2ac if sin cos are the roots of $ax^2 + bx + c = 0$ [EAMCET 2005]

1) E_1 is true, E_2 is true 2) E_1 is true, E_2 is false

3) E_1 is false, E_2 is true 4) E_1 is false, E_2 is false

Ans: 1

Sol. E_1 : 1 is a root of $ax^2 + bx + c = 0$

$\Rightarrow a(1)^2 + b(1) + c = 0$

$\Rightarrow a + b + c = 0$

E_2 : $\sin\theta + \cos\theta = -\frac{b}{a}$, $\sin\theta\cos\theta = c/a$

$(\sin\theta + \cos\theta)^2 = \left(\frac{-b}{a}\right)^2$

$1 + 2\sin\theta\cos\theta = \frac{b^2}{a^2} \Rightarrow 1 + 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2}$

$\Rightarrow b^2 - a^2 = 2ac$

$\therefore E_1$ is true, E_2 is true.

10. The set of all solutions of the inequation $x^2 - 2x + 5 \leq 0$ is [EAMCET 2004]

1) $R - (-\infty, -5)$ 2) $R - (5, \infty)$ 3) ϕ 4) $R - (-\infty, -4)$

Ans: 3

Sol. $x^2 - 2x + 5 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

$\Rightarrow x^2 - 2x + 5 > 0 \forall x \in R$

$\therefore$ $x^2 - 2x + 5 \leq 0$ has no real solution.

11 If x-2 is a common factor of the expressions $x^2 + ax + b$ and $x^2 + cx + d$, then $\frac{b-d}{c-a} =$

[EAMCET 2004]

1) -2 2) -1 3) 1 4) 2

Ans:4

Sol. $\mathbf{x-2}$ is a common factor of $\mathbf{x^2+ax+b}$ and $\mathbf{x^2+cx+d}$

$\Rightarrow \mathbf{4+2a+b=0,\ \ 4+2c+d=0}$

$\Rightarrow \mathbf{2a+b=2c+d}$

$\Rightarrow \mathbf{b-d=2(c-a)}$

$\Rightarrow \mathbf{\frac{b-d}{c-a}=2}$

12. The solution set contained in R of the inequation $3^x+3^{1-x}-4<0$ is [EAMCET 2003]

1) (1, 3) 2) (0, 1) 3) (1, 2) 4) (0, 2)

Ans: 2

Sol. $\mathbf{3^x+\frac{3}{3^x}-4<0}$

$\mathbf{3^{2x}+3-4.3^x<0}$

$\mathbf{(3^x-1)(3^x-3)<0}$

$\mathbf{1<3^x<3}$

$\therefore$ The solution set is $\mathbf{(0,1)}$

13. The minimum value of $2x^2+x-1$ is [EAMCET 2003]

1) 1/4 2) 3/2 3) -9/8 4) 9/4

Ans. 3

Sol. Minimum value of $\mathbf{ax^2+bx+c}$ is $\mathbf{\frac{4ac-b^2}{4a}}$

$\therefore$ Minimum value of $\mathbf{2x^2+x-1}$ is $\mathbf{\frac{4(2)(-1)-1}{4(2)}=-\frac{9}{8}}$

14. If the equations $x^2+ax+b=0$ and $x^2+bx+a=0$ $(a\neq b)$ have a common root then a+b= [EAMCET 2002]

1) -1 2) 2 3) 3 4) 4

Ans: 1

Sol. Let $\mathbf{'\alpha'}$ be a common root of $\mathbf{x^2+ax+b=0}$ and $\mathbf{x^2+bx+a=0}$

$\Rightarrow \mathbf{\alpha^2+a\alpha+b=0}$ -(1)

$\mathbf{\alpha^2+b\alpha+a=0}$ $\mathbf{-(2)}$ -(1)

Solve (1) & (2)

(1) – (2) $\Rightarrow$ $\mathbf{(a-b)\alpha+(b-a)=0}$

$\Rightarrow \mathbf{\alpha=1}$

$\mathbf{1+a+b=0}$

$\therefore$ $\mathbf{a+b=-1}$

15. If ‘3’ is a root of $x^2+kx-24=0$ it is also root of [EAMCET 2002]

1) $x^2+5x+k=0$ 2) $x^2+kx+24=0$ 3) $x^2-kx+6=0$ 4) $x^2-5x+k=0$

Ans: 2

Sol. **Put $x = 3$**

$\mathbf{9 + 3k - 24 = 0 \quad \Rightarrow K = 5}$

Put K=5 in options.

$\therefore$ 3 is also a root of $\mathbf{x^2 - kx + 6 = 0}$

$$\mathbf{x^2 - 5x + 6 = 0}$$

16. If $\alpha, \ \beta$ are the roots of $x^2 + bx + c = 0$ and $\alpha + h, \ \beta + h$ are the roots of $x^2 + qx + r = 0$ then h = [EAMCET 2001]

1) b+q 2) b-q 3) ½ $(b+q)$ 4) ½ $(b-q)$

Ans: 4

Sol. $\boldsymbol{\alpha, \beta}$ are the roots of $\mathbf{x^2 + bx + c = 0}$

$\Rightarrow \boldsymbol{\alpha + \beta} = \mathbf{-b}$

$\boldsymbol{\alpha + h, \beta + h}$ are the roots of $\mathbf{x^2 + qx + r = 0}$

$\Rightarrow (\boldsymbol{\alpha} + \mathbf{h}) + (\boldsymbol{\beta} + \mathbf{h}) = \mathbf{-q}$

$\boldsymbol{\alpha + \beta} + \mathbf{2h = -q}$

$\mathbf{-b + 2h = -q}$

$\Rightarrow \mathbf{h = \frac{1}{2}(b - q)}$

17. If $20^{3-2x^2} = \left(40\sqrt{5}\right)^{3x^2-2}$ then x = [EAMCET 2001]

1) $\pm\sqrt{\frac{13}{2}}$ 2) $\pm\sqrt{\frac{12}{13}}$ 3) $\pm\sqrt{\frac{4}{5}}$ 4) $\pm\sqrt{\frac{5}{4}}$

Ans: 2

Sol. $\mathbf{(20)^{3-2x^2} = \left[20x2\sqrt{5}\right]^{3x^2-2}}$

$$\mathbf{= \left[20\sqrt{20}\right]^{3x^2-2}}$$

$$\mathbf{= \left[(20)^{3/2}\right]^{3x^2-2}}$$

$\mathbf{(20)^{3-2x^2} = (20)^{\frac{9x^2-6}{2}}}$

$\mathbf{3 - 2x^2 = \frac{9x^2 - 6}{2}}$

$\mathbf{6 - 4x^2 = 9x^2 - 6}$

$\mathbf{13x^2 = 12}$

$\mathbf{x = \pm\sqrt{\frac{12}{13}}}$

18. If $\alpha, \ \beta$ are the roots of $9x^2 + 6x + 1 = 0$ then the equation with the roots $1/\alpha, \ \ 1/\beta$ is [EAMCET 2000]

1) $2x^2 + 3x + 18 = 0$ 2) $x^2 + 6x - 9 = 0$ 3) $x^2 + 6x + 9 = 0$ 4) $x^2 - 6x + 9 = 0$

Ans: 3

Sol $\mathbf{f\left(\frac{1}{x}\right)=0}$

$\Rightarrow 9\left(\frac{1}{x}\right)^2+6\left(\frac{1}{x}\right)+1=0$

$\mathbf{x^2+6x+9=0}$

19. The equation formed by decreasing each root of $ax^2+bx+c=0$ by 1 $2x^2+8x+2=0$ is then [EAMCET 2000]

1) a = -b 2) b = -c 3) c = -a 4) b=a+c

Ans: 2

Sol. The equation formed by decreasing each root of $\mathbf{ax^2+bx+c=0}$ by 1 is $\mathbf{2x^2+8x+2=0}$

$\Rightarrow$ The equation formed by increasing each root of $\mathbf{2x^2+8x+2=0}$ is $\mathbf{ax^2+bx+c=0}$

$\therefore \mathbf{ax^2+bx+c=2(x-1)^2+8(x-1)+2}$

$\mathbf{ax^2+bx+c=2x^2+4x-4}$

$\therefore \mathbf{\frac{a}{2}=\frac{b}{4}=\frac{c}{-4}\Rightarrow 2a=b=-c}$

20. If (3 + i) is a root of the equation $x^2+ax+b=0$ then a = [EAMCET 2000]

1) 3 2) -3 3) 6 4) -6

Ans: 4

Sol. If $\mathbf{3+i}$ is one root of the given equation then the other root be 3 – i

Sum of the roots of $\mathbf{x^2+ax+b=0}$ is –a

$\mathbf{\Rightarrow 3+i+3-i=-a}$

$\therefore \mathbf{a=-6}$