6. PARTIAL FRACTIONS

PREVIOUS EAMCET BITS

1. For $|x|<1$, the constant term in the expansion of $\frac{1}{(x-1)^2(x-2)}$ is **[EAMCET 2009]**

1) 2 2) 1 3) 0 4) $-\frac{1}{2}$

Ans :4

Sol: $\frac{1}{(x-1)^2(x-2)}=\frac{1}{(1-x)^2(-2)\left[1-\frac{x}{2}\right]}$

$=-\frac{1}{2}\left(1-x^2\right)\left(1-\frac{x}{2}\right)^{-1}$

$=-\frac{1}{2}\left[1+2x+3x^2+....\right]\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+...\right]$

$\therefore$ constant = $-\frac{1}{2}$

2. If $\frac{x^2+x+1}{x^2+2x+1}=A+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ then A – B = **[EAMCET 2008]**

1) 4c 2) 4c + 1 3) 3c 4) 2c

Ans: 4

Sol: $x^2+x+1=A(x+1)^2+B(x+1)C$

Put x = - 1 comparing coefficient of x^2

c = 1 A = 1 , put x = 0

A + B + C = 1

1 + B +1 = 1 $\Rightarrow$ B = – 1

$A-B=1-(-1)=2\Rightarrow 2C$

3. If $\frac{3x}{(x-a)(x-b)}=\frac{2}{x-a}+\frac{1}{x-b}$, then a : b is equal to **[EAMCET 2007]**

1) 1 : 2 2) –2:1 3) 1 : 3 4) 3 : 1

Ans: 2

Sol: $3x=2(x-b)+(x-a)$

Put x = 0

$0=-2b-a$; $a=-2b$

$\frac{a}{b}=\frac{-2}{1}$

$\therefore a:b=-2:1$

4. If $\frac{3x+2}{(x+1)(2x^2+3)}=\frac{A}{x+1}+\frac{Bx+C}{2x^2+3}$ then $A+C-B$ is equal to **[EAMCET 2006]**

1) 0 2) 2 3) 3 4) 5

Ans: 2

Sol: $3x+2=A(2x^2+3)+(Bx+C)(x+1)$

Put x = – 1

$-1=5A \Rightarrow A=-\frac{1}{5}$

Put $x=0 \Rightarrow 2=3A+C$

$2=-\frac{3}{5}+C \Rightarrow C=\frac{13}{5}$

Comparing coefficient of x^2

$2A+B=0 \Rightarrow -\frac{2}{5}+B=0 \Rightarrow B=\frac{2}{5}$

$A+C-B=-\frac{1}{5}+\frac{13}{5}-\frac{2}{5}=2$

5. If $\frac{x^3}{(2x-1)(x+2)(x-3)}=A+\frac{B}{2x-1}+\frac{C}{x+2}+\frac{D}{x-3}$, then A is equal to **[EAMCET 2005]**

1) $\frac{1}{2}$ 2) $-\frac{1}{50}$ 3) $-\frac{8}{25}$ 4) $\frac{27}{25}$

Ans: 1

Sol: $A=\frac{\text{Coefficient of } x^3 \text{ in Nr}}{\text{Coefficient } x^3 \text{in Dr}}$

$A=\frac{1}{2}$

6. If $\frac{x+1}{(2x-1)(3x+1)}=\frac{A}{2x-1}+\frac{B}{3x+1}$, then $16A+9B$ is equal to **[EAMCET 2004]**

1) 4 2) 5 3) 6 4) 8

Ans: 3

Sol: $x+1=A(3x+1)+B(2x-1)$

Put $x=\frac{1}{2} \Rightarrow \frac{3}{2}=A\left(\frac{5}{2}\right) \Rightarrow A=\frac{3}{5}$

Put $x=-\frac{1}{3}\Rightarrow\frac{2}{3}=B\left(\frac{-5}{3}\right)\Rightarrow B=-\frac{2}{5}$

$\therefore 16A+9B=16\left(\frac{3}{5}\right)+9\left(\frac{-2}{5}\right)=6$

7. Let a, b and c be such that $\frac{1}{(1-x)(1-2x)(1-3x)}=\frac{a}{1-x}+\frac{b}{1-2x}+\frac{c}{1-3x}$ then $\frac{a}{1}+\frac{b}{3}+\frac{c}{5}$ is equal to **[EAMCET 2003]**

1) $\frac{1}{15}$ 2) $\frac{1}{6}$ 3) $\frac{1}{5}$ 4) $\frac{1}{3}$

Ans: 1

Sol: $1=\ a(1-2x)(1-3x)+b(1-x)(1-3x)+c(1-x)(1-2x)$

Put x = 1

$1=a(-1)(-2)\ \Rightarrow a=\frac{1}{2}$

Put $x=\frac{1}{2}\Rightarrow 1=b\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\ \Rightarrow b=-4$

Put $x=\frac{1}{3}\Rightarrow 1=c\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\Rightarrow c=\frac{9}{2}$

Now $\frac{a}{1}+\frac{b}{3}+\frac{c}{5}=\frac{1}{2}-\frac{4}{3}+\frac{9}{2.5}=\frac{1}{15}$

8. If $\frac{1-x+6x^2}{x-x^3}=\frac{A}{x}+\frac{B}{1-x}+\frac{C}{1+x}$, then A is equal to **[EAMCET 2002]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol: Given $1-x+6x^2=A(1-x^2)+Bx(1+x)+Cx(1-x)$

Put , $x=0,$ then $A=1$

9. If $\frac{x-4}{x^2-5x-2k}=\frac{2}{x-2}-\frac{1}{x+k}$, then k = **[EAMCET 2001]**

1) -3 2) -2 3) 2 4) 3

Ans: 1

Sol: $\frac{x-4}{x^2-5x-2k}=\frac{2(x+k)-(x-2)}{(x-2)(x+k)}$

$\frac{x-4}{x^2-5x-2k}=\frac{x+2k+2}{x^2+(k-2)x-2k}$

Comparing coefficient of x in Dr

$k-2=-5 \qquad \therefore k=-3$

10. If $\frac{x^2+5}{\left(x^2+2\right)^2}=\frac{1}{x^2+2}+\frac{k}{\left(x^2+2\right)}$, then k = **[EAMCET 2000]**

1) 1 2) 2 3) 3 4) 4

Ans: 3

Sol: $x^2+5=\left(x^2+2\right)+k$

$2+k=5 \qquad \therefore k=3$