3. ALGEBRA OF MATRICES

PREVIOUS EAMCET BITS

1 If one roots of $\begin{vmatrix} 3 & 5 & x \\ 7 & x & 7 \\ x & 5 & 3 \end{vmatrix} = 0$ is – 10, then the other roots are **[EAMCET 2009]**

1) 3, 7 2) 4, 7 3) 3, 9 4) 3, 4

Ans: 1

Sol: $3(3x-35)-5(21-7x)+x(35-x^2)=0$

$\Rightarrow x^3-79x+210=0$

Verify S_1 ; $S_1 = 0$

$-10+\alpha+\beta=0$

$\alpha+\beta=10$

$\therefore$ Roots are 3, 7

2. If x, y, z are all positive and are the p^{th}, q^{th} and r^{th} terms of a geometric progression respectively, then the value of the determent $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} =$ **[EAMCET 2009]**

1) log(xyz) 2) $(p-1)(p-1)(r-1)$ 3) pqr 4) 0

Ans: 4

Sol: Let $x = AR^{p-1}, y = AR^{q-1}, Z = AR^{r-1}$

$\therefore \text{Log}x = \text{Log}A + (p-1)\text{Log}R$

$\text{Log}y = \text{Log}A + (q-1)\text{Log}R$

$\text{Log}z = \text{Log}A + (r-1)\text{Log}R$

$\begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix} = 0$

3. If $\begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix}$ has no inverse, then the real value of x is **[EAMCET 2009]**

1) 2 2) 3 3) 0 4) 1

Ans: 4

Sol: Det A = 0 $\Rightarrow x = 1$

4. If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t) = t^2 - 3t + 7$ then $f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} =$ **[EAMCET 2008]**

1) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ 2) $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ 3) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ 4) $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$

Ans: 2

Sol: $A^2 = \begin{bmatrix}1 & -2\\4 & 5\end{bmatrix}\begin{bmatrix}1 & -2\\4 & 5\end{bmatrix} = \begin{bmatrix}-7 & -12\\24 & 17\end{bmatrix}$

$f(A) = \begin{bmatrix}3 & 6\\-12 & -9\end{bmatrix} = A^2 - 3A + 7I + \begin{bmatrix}3 & 6\\-12 & -9\end{bmatrix}$

$= \begin{bmatrix}-7 & -12\\24 & 17\end{bmatrix} - 3\begin{bmatrix}1 & -2\\4 & 5\end{bmatrix} + 7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}3 & 6\\-12 & -9\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

5. The inverse of the matrix $\begin{bmatrix}7 & -3 & 3\\-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}$ **[EAMCET 2008]**

1) $\begin{bmatrix}1 & 1 & 1\\3 & 4 & 3\\3 & 3 & 4\end{bmatrix}$ 2) $\begin{bmatrix}1 & 3 & 1\\4 & 3 & 8\\3 & 4 & 1\end{bmatrix}$ 3) $\begin{bmatrix}1 & 1 & 1\\3 & 3 & 4\\3 & 4 & 3\end{bmatrix}$ 4) $\begin{bmatrix}1 & 3 & 3\\1 & 4 & 3\\1 & 3 & 4\end{bmatrix}$

Ans:

Sol: $AA^{-1} = I$

$\begin{bmatrix}7 & -3 & 3\\-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 3\\1 & 4 & 3\\1 & 3 & 4\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

6. $\begin{vmatrix}a-b-c & 2a & 2a\\2b & b-c-a & 2b\\2c & 2c & c-a-b\end{vmatrix} =$ **[EAMCET 2008]**

1) 0 2) a + b + c 3) $(a + b + c)^2$ 4) $(a + b + c)^3$

Ans: 4

Sol: Put a = 1, b = 1, c = 1

$\begin{vmatrix}-1 & 2 & 2\\2 & -1 & 2\\2 & 2 & -1\end{vmatrix} = -1(-3) - 2(-6) + 2(6) = 27$

$(a+b+c)^3 = (1+1+1)^3 = 27$

7. If $\begin{bmatrix}1 & 2 & x\\4 & -1 & 7\\2 & 4 & -6\end{bmatrix}$ is a singular matrix, then x = **[EAMCET 2007]**

1) 0 2) 1 3) – 3 4) 3

Ans: 3

Sol: det A = 0

$1(6-28)-2(-24-14)+x(16+2)=0$

$-22+76+18x=0$

$\Rightarrow x=-3$

8. If A is a square matrix such that A (Adj A) = $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ then det (Adj A) = **[EAMCET 2007]**

1) 4 2) 16 3) 64 4) 256

Ans: 2

Sol: $A(\text{Adj}A)=|A|I$

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}=\begin{pmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{pmatrix}$$

|A| = 4

$\det(\text{Adj}\,A)=(\det A)^{n-1}$

$=(4)^{3-1}$

= 16

9. The number of nontrivial solutions of the system $x-y+z=0, x+2y-z=0, 2x+y+3z=0$ is **[EAMCET 2007]**

1) 0 2) 1 3) 2 4)3

Ans: 1

Sol: Write given system of equation is matrix form AX = B $\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now $|A|=\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}=9$

Since $|A|\neq 0$. So given system of equation is has only trivial solution, so there is no non-trivial solution.

10. $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then A^3-4A^2-6A is equal to **[EAMCET 2006]**

1) 0 2) A 3) – A 4) I

Ans: 3

Sol: $A^2=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$$A^3 \; A^2.A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 41 & 42 & 42 \\ 42 & 41 & 42 \\ 42 & 42 & 41 \end{bmatrix}$$

$$A^3 - 4A^2 - 6A = \begin{bmatrix} 41 & 42 & 42 \\ 42 & 41 & 42 \\ 42 & 42 & 41 \end{bmatrix} - 4\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 6\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & -2 \\ -2 & -1 & -2 \\ -2 & -2 & -1 \end{bmatrix} = -A$$

11. $\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix} =$ **[EAMCET 2006]**

1) 0 2) 1 3) 4 loge 4) 5 loge

Ans: 1

Sol: $\begin{vmatrix} \log e & 2\log e & 3\log e \\ 2\log e & 3\log e & 4\log e \\ 3\log e & 4\log e & 5\log e \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$C_2 - C_1$ and $C_3 - C_2$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0$$

12. If A is an invertible matrix of order n, then the determinant of adjA is equal to **[EAMCET 2006]**

1) $|A|^n$ 2) $|A|^{n+1}$ 3) $|A|^{n-1}$ 4) $|A|^{n+2}$

Ans: 3

Sol: Since A is invertiable matrix of order n, then the determinant of adjA = $|A|^{n-1}$

13. $m\begin{bmatrix} -3 & 4 \end{bmatrix} + n\begin{bmatrix} 4 & -3 \end{bmatrix} = \begin{bmatrix} 10 & -11 \end{bmatrix} \Rightarrow 3m + 7n =$ **[EAMCET 2005]**

1) 3 2) 5 3) 10 4) 1

Ans: 1

Sol: $-3m + 4n = 10$ …………..(1)

$4m - 3n = -11$ ………….(2)

Solve (1) & (2) we get

m = – 2, n = 1

$\therefore 3m + 7n = -6 + 7 = 1$

14. $\text{Adj}\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix} \Rightarrow [a \ \ b] =$ **[EAMCET 2005]**

1) [-4 1] 2) [–4 –1] 3) [4 1] 4) [4 –1]

Ans: 3

Sol: Give that $\text{adj}\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$

Cofactors of $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -2 \\ 4 & 1 & -2 \\ -2 & 0 & 1 \end{bmatrix}$

$$\text{Adj}\begin{bmatrix} 1 & 0 & 2 \\ -1 & 0 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & -2 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$$

$\Rightarrow a = 4, b = 1$

$[a \ \ b] = [4 \ 1]$

15 $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow A^3 - A^2 =$ **[EAMCET 2005]**

1) 2A 2) 2I 3) A 4) I

Ans: 1

Sol: $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix}$

$A^3 - A^2 = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix} = 2A$

16. Match the following elements of $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 2 \\ 3 & -4 & 6 \end{bmatrix}$ with their cofactors and choose the correct answer

[EAMCET 2004]

Element	**Cofactor**
I) – 1	a) –2
II) 1	b) 32
III) 3	c) 4
IV) 6	d) 6
	e) – 6

1) b, d, a, c 2) b, d, c, a 3) d, b, a, c 4) d, a, b, c

Ans: 3

Sol: Cofactor of $-1 = (-1)^{1+2}\begin{vmatrix} 0 & 2 \\ 3 & 6 \end{vmatrix} = +6$

Cofactor of $1 = (-1)^{1+1}\begin{vmatrix} 4 & 2 \\ -4 & 6 \end{vmatrix} = 32$

Cofactor of $3 = (-1)^{3+1}\begin{vmatrix} -1 & 0 \\ 4 & 2 \end{vmatrix} = -2$

Cofactor of $6 = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 0 & 4 \end{vmatrix} = 4$

17. $\det\begin{bmatrix} 1990 & 1991 & 1992 \\ 1991 & 1992 & 1993 \\ 1992 & 1993 & 1994 \end{bmatrix} =$ **[EAMCET 2004]**

1) 1992 2) 1993 3) 1994 4) 0

Ans: 4

Sol: $C_2 - C_1$ and $C_3 - C_2$

$$\begin{vmatrix} 1990 & 1 & 1 \\ 1991 & 1 & 1 \\ 1992 & 1 & 1 \end{vmatrix} = 0$$

18. The rank of $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ is **[EAMCET 2004]**

1) 0 2) 1 3) 2 4) 3

Ans: 4

Sol: $\det A = 1(1+1) + 1(1-1) + 1(1+1)$

$= 4$

$\det A \neq 0$

$\therefore$ Rank of A = 3

19. If p + q + r = 0 and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ then k = **[EAMCET 2003]**

1) 0 2) abc 3) pqr 4) a + b + c

Ans: 3

Sol: Put p = 1, q = 1, r = – 2, a = 1, b = 1, c = 2

$$\begin{vmatrix} 1 & 1 & -4 \\ 2 & -2 & +1 \\ -2 & 2 & 1 \end{vmatrix} = K\begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$\Rightarrow -8 = K(4) \Rightarrow K = -2$

$\therefore K = pqr$

20. If $\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos 2B \\ \sin A & \cos A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix} = 0$ then B = **[EAMCET 2003]**

1) $(2n+1)\frac{\pi}{2}$ 2) $n\pi$ 3) $(2n+1)\pi$ 4) $2n\pi$

Ans: 1

Sol: $\cos^2(A+B) + \sin^2(A+B) + \cos 2B = 0$

$1 + \cos 2B = 0$

$2\cos^2 B = 0$

$\cos B = 0 \Rightarrow B = (2n+1)\frac{\pi}{2}, n \in Z$

21. The number of solutions of the system equations $2x + y - z = 7, x - 3y + 3z = 1, x + 4y - 3z = 5$ is

1) 3 2) 2 3) 1 4) 0 **[EAMCET 2003]**

Ans: 3

Sol: Given $2x + y - z = 7$ ………….(1)

$x - 3y + 2z = 1$…………(2)

$x + 4y - 3z = 5$ ………….(3)

From (1) & (2) we get

$5x - y = 15$ ………..…..(4)

From (1) & (3) we get

$5x - y = 16$ ……………..(5)

(4) & (5) are parallel

$\therefore$ solution does not exist.

22. If A, B are square matrices of order 3, A is non-singular and AB = 0, then B is a **[EAMCET 2002]**

1) Null matrix 2) Non –singular matrix

3) Singular matrix 4) Unit matrix

Ans: 1 and 3

Sol: Given AB = 0

$\Rightarrow$ B = 0 $[\because A$ is non-singular$]$ (or)

|AB| = 0

$|A||B| = 0 \Rightarrow |B| = 0$ $[\because A$ is non-singular $|A| \neq 0]$

$\therefore$ B is null matrix (or) singular matrix

23. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ then, det A = **[EAMCET 2002]**

1) 2 2) 3 3) 4 4) 5

Ans: 1

Sol: $\det A = 1(1-0)+1(4-3)$

$= 2$

24. If $x^2+y^2+z^2 \neq 0, x = cy+bz, y = ax+cx$ and $z = bx+ay$, then $a^2+b^2+c^2+2abc =$

[EAMCET 2002]

1) 2 2) a + b + c 3) 1 4) ab + bc + ca

Ans: 3

Sol: Given $x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow 1(1-a^2)+c(-c-ab)-b(ac+b) = 0$

$\therefore a^2+b^2+c^2+2abc = 1$

25. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $\left(B^{-1}A^{-1}\right)^{-1} =$ **[EAMCET 2001]**

1) 2) 3) 4)

Sol: $\left(B^{-1}A^{-1}\right)^{-1} = \left(A^{-1}\right)^{-1}\left(B^{-1}\right)^{-1} = AB$

$= \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+2 & -2+0 \\ 0+2 & 3+0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$

26. A square matrix (a_{ij}) in which $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = k$ (constant) for i = j is

[EAMCET 2001]

1) Unit matrix 2) Scalar matrix 3) Null matrix 4) Diagonal matrix

Ans: 2

Sol: $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{bmatrix} \therefore a_{ij} = 0 \text{ for } i \neq j \\ a_{ij} = k \text{ for } i = j \end{bmatrix}$

It is a scalar matrix

27. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}, KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b, are respectively **[EAMCET 2001]**

1) $-6,-12,-18$ 2) $-6,4,9$ 3) $-6,-4,9$ 4) $-6,12,18$

Ans : 3

Sol: $KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$-4k = 24$	$2k = 3a$	$3k = 2b$
$k = -6$	$-12 = 3a$	$-18 = 2b$
	$a = -4$	$b = -9$

28. If A and B are two square matrices such that $= -A^{-1}BA$ then $\left(A^2 + B^2\right) =$ **[EAMCET 2000]**

1) 0 2) $A^2 + B^2$ 3) $A^2 + 2AB + B^2$ 4) $A + B$

Ans: 2

Sol: Given $B = A^{-1}BA$

$$\begin{aligned}
(A+B)^2 &= A^2 + AB + BA + B^2 \\
&= A^2 + A\left(-A^{-1}BA\right) + BA + B^2 \\
&= A^2 - AA^{-1}BA + BA + B^2 \\
&= A^2 - IBA + BA + B^2 \\
&= A^2 - BA + BA + B^2 \\
&= A^2 + B^2
\end{aligned}$$

29 If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then the determinant $A^2 - 2A$ is **[EAMCET 2000]**

1) 5 2) 25 3) –5 4) –25

Sol: $A^2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix}$$

$$A^2 - 2A = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -2 & 5 \end{bmatrix}$$

$$\left|A^2 - 2A\right| = 25 - 0 = 25$$

30. If ‘d’ is the determinant of a square matrix A of order n, then the determinant of its adjoint is

1) d^n 2) d^{n-1} 3) d^{n-2} 4) d **[EAMCET 2000]**

Ans: 2

Sol: |A| = d

$|\text{adj}A| = |A|^{n-1}$

$= d^{n-1}$

31. If $a \neq 6$, b, c satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then abc = **[EAMCET 2000]**

1) a + b + c 2) 0 3) b^3 4) ab – b –c

Ans:

Sol: $a\left(b^2 - ac\right) - 2b\left(3b - 4c\right) + 2c\left(3a - 4b\right) = 0$

$ab^2 - a^2c - 6b^2 + 8bc + 6ac - 8bc = 0$

$ab^2 - a^2c - 6b^2 + 6ac = 0$

$b^2\left(a - 6\right) - ac\left(a - 6\right) = 0$

$\left(b^2 - ac\right)\left(a - 6\right) = 0$

$b^2 - ac = 0 \quad [\because a \neq 6]$

$b^2 = ac$

$b^2.b = abc$

$\therefore abc = b^3$