HYPERBOLA

PREVIOUS EAMCET BITS

1. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points (x_i, y_i), for i =1, 2, 3 and 4, then $y_1 + y_2 + y_3 + y_4 =$ **[EAMCET 2009]**

1) 0 2) c 3) a 4) c^4

Ans: 1

Sol. $x = \frac{c^2}{y} \Rightarrow \frac{c^4}{y^2} + y^2 = a^2$

$\Rightarrow y^4 - a^2y^2 + c^4 = 0$

$\Rightarrow y_1 + y_2 + y_3 + y_4 = 0$

2. The midpoint of the chord 4x – 3y = 5 of the hyperbola $2x^2 - y^3 = 12$ is **[EAMCET 2009]**

1) $\left(0, -\frac{5}{3}\right)$ 2) (2, 1) 3) $\left(\frac{5}{4}, 0\right)$ 4) $\left(\frac{11}{4}, 2\right)$

Ans: 2

Sol. Write $S_1 = S_{11}$ and Compare

3. The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ **[EAMCET 2008]**

1) 4 2) 6 3) 8 4) 10

Ans: 3

Sol. $x^2 - 3y^2 - 4x - 6y - 11 = 0$

$\Rightarrow (x^3 - 4x + 4) - 3(y^2 + 2y + 1) - 12 \Rightarrow (x-2)^2 - 3(y+1)^2 = 12$

$\Rightarrow \frac{(x-2)^2}{12} - \frac{(y+1)^2}{4} = 1 \Rightarrow a^2 = 12, b^2 = 4 \Rightarrow e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{12+4}}{4} = \frac{4}{4} = 1$

$b^2 = a^2(e^1 - 1) \Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 - 1 = \frac{1}{3} \Rightarrow e^2 = \frac{4}{3} \Rightarrow e = \frac{2}{\sqrt{3}}$

Distance between the foci = $2ae = 2\sqrt{12}\left(2/\sqrt{3}\right) = 8$

4. If the line $\ell x + my = 1$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\frac{a^2}{\ell^2} - \frac{b^2}{m^2} =$ **[EAMCET 2007]**

1) $a^2 - b^2$ 2) $a^2 + b^2$ 3) $\left(a^2 + b^2\right)^2$ 4) $\left(a^2 - b^2\right)^2$

Ans: 3

Sol. $\frac{a^2}{\ell^2} - \frac{b^2}{m^2} = \frac{\left(a^2+b^2\right)^2}{n^2}$

$$\Rightarrow \frac{a^2}{\ell^2} - \frac{b^2}{m^2} = \left(a^2 + b^2\right)^2$$

5. If the eccentricity of a hyperbola is $\sqrt{3}$, then the eccentricity of its conjugate hyperbola is **[EAMCET 2006]**

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\sqrt{\frac{3}{2}}$ 4) $2\sqrt{3}$

Ans: 3

Sol. $\frac{1}{e^2} + \frac{1}{(e')^2} = 1$

$\frac{1}{3} + \frac{1}{(e')^2} = 1 \Rightarrow \frac{1}{(e')^2} = 1 - \frac{1}{3} = \frac{2}{3}$

$e^1 = \sqrt{\frac{3}{2}}$

6. The product of the lengths of perpendicular drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes is **[EAMCET 2003]**

1) $\frac{1}{2}$ 2) $\frac{2}{3}$ 3) $\frac{3}{2}$ 4) 2

Ans: 2

Sol. $\frac{x^2}{2} \frac{-y^2}{1} = 1$

Product of the length of perpendiculars from any point on the hyperbola to its asymptotes

$\frac{a^2b^2}{a^2 + b^2} = \frac{2}{3}$

7. If e and e' are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola $5x^2 - 4y^2 = 45$ respectively, then $ee' =$ **[EAMCET 20002]**

1) 9 2) 5 3) 4 4) 1

Ans: 4

Sol. Ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\Rightarrow e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{9-5}{9}} = \frac{2}{3}$

Hyperbola $\frac{x^2}{9} - \frac{y^2}{45/4} = 1$

$\Rightarrow e^1 = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{9 + \frac{45}{4}}{9}} = \frac{3}{2}$

$e.e^{1} = \frac{2}{3}.\frac{3}{2} = 1$

8. **[EAMCET 2002]**

1) 2) 3) 4)

Ans:

Sol.

9. The equation $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$ represents **[EAMCET 2001]**

1) A circle 2) A parabola 3) An ellipse 4) A hyperbola

Ans: 2

Sol. $h^2 - ab = 16 - 16 = 0$

10. The curve represented by x = 2(cost + sint) and y = 5 (cost – sint) is **[EAMCET 2000]**

1) a circle 2) a parabola 3) an ellipse 4) a hyperbola

Ans: 3

Sol. $\frac{x}{2} = \left(\cos t + \sin t\right) = \frac{y}{5} = \cos t = \sin t$

$\frac{x^2}{4} + \frac{y^2}{25} = 2$

✿ ✿ ✿