

7. EXPONENTIAL SERIES AND LOGARITHMIC SERIES

PREVIOUS EAMCET BITS

1. $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \dots \Rightarrow 2a_1 + 2^3a_3 + 2^5a_5 + \dots =$ **[EAMCET 2009]**
 1) e 2) e^{-1} 3) 1 4) 0

Ans:

Sol: $e^{-2x} + e^{2x}$
 $= 2 \left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \dots \right]$
 $\Rightarrow a_1 = 0, a_3 = 0, a_5 = 0, \dots$ and so on.

2. $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \dots =$ **[EAMCET 2008]**
 1) $2\log_e^2 - 2$ 2) $2 - 2\log_e^2$ 3) $2\log_e^4$ 4) \log_e^4

Ans: 2

Sol: $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \dots = \sum_{n=1}^{\infty} \frac{1}{n(2n+1)}$
 $= 2 \sum_{n=1}^{\infty} \frac{1}{2n(2n+1)} = 2 \sum_{n=1}^{\infty} \left[\frac{1}{2n} - \frac{1}{2n+1} \right]$
 $= 2 \left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots \right]$
 $= 2 - 2 \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right] = 2 - 2\log_e^2$

3. The coefficient of x^k in the expansion of $\frac{1-2x-x^2}{e^{-x}}$ is **[EAMCET 2007]**
 1) $\frac{1-k-k^2}{k!}$ 2) $\frac{k^2+1}{k!}$ 3) $\frac{1-k}{k!}$ 4) $\frac{1}{k!}$

Ans : 1

Sol: Now $\frac{1-2x-x^2}{e^{-x}} = (1-2x-x^2)e^x$
 $= [1-2x-x^2] \left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^k}{k!}+\dots \right]$
 $= \left(1+x+\frac{x^2}{2!}+\dots+\frac{x^k}{k!}+\dots \right) - 2 \left[x+\frac{x^3}{2!}+\dots+\frac{x^k}{(k-1)!}+\frac{x^{k+1}}{k!}+\dots \right]$
 $- \left[x^2+x^3+\frac{x^4}{4!}+\dots+\frac{x^k}{(k-2)!}+\frac{x^{k+1}}{(k-1)!}+\dots \right]$

\therefore coefficient of x^k is $\frac{1}{k!} - \frac{2}{(k-1)!} - \frac{1}{(k-2)!}$

$$\frac{1}{k!} - \frac{2k}{k!} - \frac{k(k-1)}{k!} = \frac{1-k-k^2}{k!}$$

4. $\frac{1}{2} - \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} - \frac{1}{4 \cdot 2^4} + \dots =$

[EAMCET 2007]

- 1) $\frac{1}{4}$ 2) $\log_3\left(\frac{3}{4}\right)$ 3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{2}{3}\right)$

Ans: 2

Sol: $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \log(1+x)$

$$\frac{1}{2} - \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^4}{4} + \dots = \log\left(1 + \frac{1}{2}\right) = \log_3\left(\frac{3}{2}\right)$$

5. The coefficient of x^n in $\frac{1-2x}{e^x}$ is

[EAMCET 2006]

- 1) $\frac{1+2n}{n!}$ 2) $(-1)^n \frac{(1+2n)}{n!}$ 3) $(-1)^n \frac{(1-2n)}{n!}$ 4) $(-1)^n \frac{(1+4n)}{n!}$

Ans :

Sol: $\frac{1-2x}{e^x} = (1-2x)e^{-x}$

$$(1-2x) \left[1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} + \dots \right]$$

Coefficient of x^n in $\frac{1-2x}{e^x}$

$$= \frac{(-1)^n}{n!} - \frac{2(-1)^{n-1}}{(n-1)!} \Rightarrow (1)^n \frac{(1-2n)}{n!}$$

6. If $|x| < 1$ and $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ then x is equal to

[EAMCET 2006]

- 1) $y + \frac{y^2}{2} + \frac{y^3}{3} + \dots$ 2) $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$
 3) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ 4) $y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \dots$

Ans:

Sol: $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$$y = \log_e(1+x) \Rightarrow 1+x = e^y$$

$$\Rightarrow 1+x = 1+y + \frac{y^2}{2!} + \dots$$

$$\therefore x = y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$$

7. $\sum_{n=1}^{\infty} \frac{2n^2 + n + 1}{n!} =$

[EAMCET 2005]

1) $2e-1$

2) $2e+1$

3) $6e-1$

4) $6e+1$

Ans: 3

Sol: Let $s = \sum_{n=1}^{\infty} \frac{2n^2 + n + 1}{n!} = \sum_{n=1}^{\infty} \left[\frac{2n^2}{n!} + \frac{n}{n!} + \frac{1}{n!} \right]$

$$= \sum_{n=1}^{\infty} \left[\frac{2}{(n-2)!} + \frac{3}{(n-1)!} + \frac{1}{n!} \right]$$

$$= 2 \left[1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \infty \right] + 3 \left[1 + \frac{1}{1!} + \frac{1}{2!} + \dots \infty \right] + \left[\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \infty \right]$$

$$= 2e + 3e + e - 1 = 6e - 1$$

8. If $|a| < 1, b = \sum_{n=1}^{\infty} \frac{a^k}{k}$, then a is equal to

[EAMCET 2005]

1) $\sum_{n=1}^{\infty} \frac{(-1)^k b^k}{k}$

2) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} b^k}{k!}$

3) $\sum_{k=1}^{\infty} \frac{(-1)^k b^k}{(k-1)!}$

4) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} b^k}{(k+1)!}$

Ans: 2

Sol: $b = \sum_{k=1}^{\infty} \frac{a^k}{k!} = \frac{a}{1} + \frac{a^2}{2} + \frac{a^3}{3} + \dots \infty$

$$b = -\log_e (1-a)$$

$$e^{-b} = 1-a \Rightarrow a = 1-e^{-b} = 1 - \left[1 - \frac{b}{1!} + \frac{b^2}{2!} - \frac{b^3}{3!} + \dots \infty \right]$$

$$\therefore a = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} b^k}{k!}$$

9. The value of the series $\therefore a = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} b^k}{k!}$

[EAMCET 2004]

1) $\cosh(x \log_e^a)$

2) $\coth(x \log_e^a)$

3) $\sinh(x \log_e^a)$

4) $\tanh(x \log_e^a)$

Ans: 3.

Sol: $\frac{e^{x \log_e^a} - e^{-x \log_e^a}}{2} \left[\because \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right]$

$$= \sinh[x \log_e^a]$$

10. Coefficient of x^{10} in the expansion of $(2+3x)e^{-x}$ is **[EAMCET 2004]**

- 1) $-\frac{26}{(10)!}$ 2) $-\frac{28}{(10)!}$ 3) $-\frac{30}{(10)!}$ 4) $-\frac{32}{(10)!}$

Ans: 2

Sol: $(2+3x)e^{-x} = (2+3x) \left[1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{x^9}{9!} + \frac{x^{10}}{10!} + \dots \right]$

\therefore Coefficient of x^{10} in the above series

$$= \frac{2}{10!} - \frac{3}{9!} = \frac{1}{10!}(2-30) \Rightarrow -\frac{28}{10!}$$

11. $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots$ is equal to **[EAMCET 2003]**

- 1) $\frac{e}{2}$ 2) $\frac{e}{3}$ 3) $\frac{e}{4}$ 4) $\frac{e}{5}$

Ans: 1

Sol:
$$= \sum_{n=1}^{\infty} \frac{1+2+3+\dots+n}{(n+1)!}$$

$$= \sum_{n=1}^{\infty} \frac{n(n+1)}{2(n+1)n(n-1)!} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$

$$= \frac{1}{2} \left[1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right] = \frac{1}{2} e = \frac{e}{2}$$

12. If $0 < y < 2^{1/3}$ and $x(y^3 - 1) = 1$, then $\frac{2}{x} + \frac{2}{3x^3} + \frac{2}{5x^5} + \dots$ is equal to **[EAMCET 2003]**

- 1) $\log\left(\frac{y^3}{2-y^3}\right)$ 2) $\log\left(\frac{y^3}{1-y^3}\right)$ 3) $\log\left(\frac{2y^3}{1-y^3}\right)$ 4) $\log\left(\frac{y^3}{1-2y^3}\right)$

Ans: 1

Sol: $y^3 - 1 = \frac{1}{x}$

$$x + \frac{x^3}{3} + \frac{x^5}{5} + \dots = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$$

$$= 2 \left[\frac{1}{x} + \frac{\left(\frac{1}{x}\right)^3}{3} + \frac{\left(\frac{1}{x}\right)^5}{5} + \dots \right] \Rightarrow 2 \times \frac{1}{2} \log\left(\frac{1+1/x}{1-1/x}\right)$$

$$= \log\left[\frac{1+y^3-1}{1-y^3+1}\right] = \log\left[\frac{y^3}{2-y^3}\right]$$

13. $1 + x \log_e^a + \frac{x^2}{2!} (\log_e^a)^2 + \frac{x^3}{3!} (\log_e^a)^3 + \dots$ ($a > 0, x \in \mathbb{R}$) is equal to **[EAMCET 2002]**

- 1) a 2) a^x 3) $a^{\log_e x}$ 4) x

Ans: 2

Sol: $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$

$$1 + \frac{x \log_e a}{1!} + \frac{(x \log_e a)^2}{2!} + \dots = e^{x \log_e a} = e^{\log_e a^x} = a^x$$

14. $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \dots$ is equal to

[EAMCET 2002]

- 1) $e^2 + e$ 2) e^2 3) $e^2 - 1$ 4) $e^2 - e$

Ans: 4

Sol:
$$\sum_{n=1}^{\infty} \frac{1+2+2^2+\dots+2^{n-1}}{n!} = \sum_{n=1}^{\infty} \frac{1(2^n-1)}{n!}$$

$$= \sum_{n=1}^{\infty} \frac{2^n-1}{n!} = \sum_{n=1}^{\infty} \frac{2^n}{n!} - \sum_{n=1}^{\infty} \frac{1}{n!}$$

$$= (e^2 - 1) - (e - 1) = e^2 - e$$

15. $\frac{2}{2!} + \frac{2+4}{3!} + \frac{2+4+6}{4!} + \dots$

[EAMCET 2001]

- 1) e 2) e^{-1} 3) e^{-2} 4) e^{-3}

Ans: 1

Sol:
$$= \sum_{n=1}^{\infty} \frac{2+4+6+\dots+2n}{(n+1)!} \Rightarrow \sum_{n=1}^{\infty} \frac{2(1+2+3+\dots+n)}{(n+1)!}$$

$$= \sum_{n=1}^{\infty} \frac{n(n+1)}{(n+1)n(n-1)!} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$

$$= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots = e$$

16. $|x| < 1$, the coefficient x^3 in the expansion of $\log(1+x+x^2)$ in ascending powers of x is

[EAMCET 2001]

- 1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $-\frac{2}{3}$ 4) $-\frac{4}{3}$

Ans: 3

Sol: We have $\log(1+x+x^2) = \log \left[\frac{(1+x+x^2)(1-x)}{1-x} \right] = \log \left[\frac{1-x^3}{1-x} \right]$

$$= \log[1-x^3] - \log[1-x] = - \left[x^3 + \frac{x^6}{2} + \frac{x^9}{3} + \dots \right] + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \right]$$

Coefficient of x^3 is $-1 + \frac{1}{3} = -\frac{2}{3}$

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