DEFINITE INTEGRATION

PREVIOUS EAMCET BITS

1. $\int_0^{\pi} \frac{1}{1+\sin x}dx =$ **[EAMCET 2009]**

1) 1 2) 2 3) – 1 4) – 2

Ans: 2

Sol: $\int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}dx$

$= \int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx \Rightarrow \int_0^{\pi} \left(\sec^2 x - \sec x \tan x\right)dx$

$= \left(\tan x - \text{sex}\right)_0^{\pi}$

$= \left[\tan \pi - \sec \pi\right] - \left[\tan 0° - \sec 0°\right]$

$= (0+1) - (0-1) = 2$

2. $\int_0^1 x^{3/2}\sqrt{1-x}dx =$ **[EAMCET 2008]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{9}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{16}$

Ans: 4

Sol: Put $x = \sin^2\theta$ then $dx = 2\sin\theta\cos\theta d\theta$

Also x = 0, 1 $\Rightarrow$ 0, π/2

$\int_0^1 x^{3/2}\sqrt{1-x}dx = \int_0^{\pi/2} \sin^3\theta\sqrt{1-\sin^2\theta}.2\sin\theta\cos\theta d\theta$

$= 2\int_0^{\pi/2} \sin^4\theta \cos^2\theta d\theta = 2 \times \frac{1}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{16}$

3. $\int_{-\pi/2}^{\pi/2} \sin|x|dx =$ **[EAMCET 2008]**

1) 0 2) 1 3) 2 4) π

Ans:3

Sol: $\int_{-\pi/2}^{\pi/2} \sin|x|dx = \int_{-\pi/2}^{0} \sin(-x)dx + \int_0^{\pi/2} \sin x dx$

$= [\cos x]_{-\pi/2}^{0} + [-\cos x]_0^{\pi/2}$

$= 1 - 0 - 0 + 1 = 2$

4. If $f(x)=\int_{-t}^{t}\frac{e^{-|x|}}{2}dx$ then $\underset{t\to\infty}{Lt} f(t)$ is **[EAMCET 2007]**

1) 1 2) 1/2 3) 0 4) – 1

Ans:1

Sol: $f(t)=\int_{-t}^{0}\frac{e^{x}}{2}dx+\int_{0}^{t}\frac{e^{-x}}{2}dx$

$=\frac{1}{2}\left[e^{x}\right]_{-t}^{0}+\frac{1}{2}\left[-e^{-x}\right]_{0}^{t}$

$=\frac{1}{2}\left[1-e^{-t}\right]-\frac{1}{2}\left[e^{-t}-1\right]=\frac{1}{2}-\frac{1}{2}e^{-t}-\frac{1}{2}e^{-t}+\frac{1}{2}$

f(t) $=1-e^{-t}$

$\underset{t\to\infty}{Lt} f(t)=\underset{x\to\infty}{Lt}\left[1-e^{-t}\right]=1-e^{-\infty}=1-0=1$

5. $\int_{0}^{2\pi}\sin^{6}x\cos^{5}x\,dx=$ **[EAMCET 2007]**

1) 2π 2) $\frac{\pi}{2}$ 3) 0 4) $-\pi$

Ans: 3

Sol: $I=\int_{0}^{2\pi}\sin^{6}x\cos^{5}x\,dx$

Let $f(x)=\sin^{6}x\cos^{5}x$

$f(2\pi-x)=f(x)$

$I=2\int_{0}^{\pi}\sin^{6}x\cos^{5}x\,dx$

$f(\pi-x)=\sin^{6}(\pi-x)\cos^{5}(\pi-x)$

$=-\sin^{6}x\cos^{5}x=-f(x)$

$\therefore I=0$

6. $\int_{0}^{\pi/2}\frac{dx}{1+\tan^{3}x}=$ **[EAMCET 2006]**

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{2}$

Ans: 3

Sol: Let $I=\int_{0}^{\pi/2}\frac{dx}{1+\tan^{3}x}$

$$= \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \ldots\ldots\ldots(1)$$

$$= \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx \ldots\ldots\ldots\ldots(2)$$

(1) + (2)

$$2I = \int_0^{\pi/2} 1.dx$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

7. $\int_{-1}^{1} \frac{\cosh x}{1 + e^{2x}} dx =$ **[EAMCET 2006]**

1) 0 2) 1 3) $\frac{e^2 - 1}{2e}$ 4) $\frac{e^2 + 2}{2e}$

Ans: 3

Sol: $\int_{-1}^{1} \frac{e^x + e^{-x}}{2\left(1 + e^{2x}\right)} dx \quad \left[\because \cosh x = \frac{e^x + e^{-x}}{2}\right]$

$$= \int_{-1}^{1} \frac{e^{2x} + 1}{2e^x\left(1 + e^{2x}\right)} dx \Rightarrow \frac{1}{2}\int_{-1}^{1} e^{-x} dx$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-x} dx \Rightarrow \frac{1}{2}\left[-e^{-x}\right]_{-1}^{1}$$

$$= -\frac{1}{2}\left[e^{-1} - e^{1}\right] \Rightarrow \frac{e^2 - 1}{2e}$$

8. $\int_0^{\pi/2} \frac{200\sin x + 100\cos x}{\sin x + \cos x} dx =$ **[EAMCET 2005]**

1) 50π 2) 25π 3) 75π 4) 150π

Ans: 3

Sol: $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + cox} dx = (a + b)\frac{\pi}{4}$

$$\int_0^{\pi/2} \frac{200\sin x + 100\cos x}{\sin x + \cos x} dx = (200 + 100)\frac{\pi}{4} = 75\pi$$

9. $\int_{0}^{\pi}\frac{\theta\sin\theta}{1+\cos^2\theta}d\theta =$ **[EAMCET 2005]**

1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^3}{3}$ 3) π^2 4) $\frac{\pi^2}{4}$

Ans: 4

Sol: Let I = $\int_{0}^{\pi}\frac{\theta\sin\theta}{1+\cos^2\theta}d\theta$

$= \int_{0}^{\pi}\frac{(\pi-\theta)\sin(\pi-\theta)}{1+\cos^2(\pi-\theta)}d\theta \Rightarrow \int_{0}^{\pi}\frac{(\pi-\theta)\sin\theta}{1+\cos^2\theta}d\theta$

$I = \pi\int_{0}^{\pi}\frac{\sin\theta}{1+\cos^2\theta}d\theta - I =$ $2I = \pi.2\int_{0}^{\pi/2}\frac{\sin\theta}{1+\cos^2\theta}d\theta$ let cosθ = 5 , - sinθdθ = dt

$I = \pi\int_{1}^{0}\frac{-dt}{1+t^2} \Rightarrow \pi\int_{0}^{1}\frac{1}{1+t^2}dt$

$= \pi\left[\tan^{-1}t\right]_0^1 \Rightarrow I = \pi\left[\frac{\pi}{4}-0\right] \Rightarrow I = \frac{\pi^2}{4}$

10. $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta =$ **[EAMCET 2004]**

1) 0 2) 1 3) 2 4) – 1

Ans: 1

Sol: Let $f(\theta) = \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)$

$f(-\theta) = \log\left(\frac{2+\sin\theta}{2-\sin\theta}\right) = -\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)$

$f(-\theta) = -f(\theta)$

$\therefore$ It is an odd function

$\therefore I = \int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta = 0$

11. $\int_{0}^{2}\frac{2x-2}{2x-x^2}dx =$ **[EAMCET 2004]**

1) 0 2) 2 3) 3 4) 4

Ans: 1

Sol: Let I = $\int_{0}^{2}\frac{2x-2}{2x-x^2}dx$

Put $2x - x^2 = t \Rightarrow (2-2x)dx = dt$

$$-\int_0^0 \frac{dt}{t} = 0$$

12. $\int_{-2}^{2} \|[x]\| dx =$ **[EAMCET 2003]**

1) 1 2) 2 3) 3 4) 4

Ans: 4

Sol: $\int_{-2}^{-1} \|[x]\| dx + \int_{-1}^{0} \|[x]\| dx + \int_{0}^{1} \|[x]\| dx + \int_{1}^{2} \|[x]\| dx$

= 2 + 1 + 0 + 1 = 4

13. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx =$ **[EAMCET 2003]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

Ans: 2

Sol: Put x = cosθ   L.L : 0 = cosθ $\Rightarrow \theta = \pi/2$

dx = - sinθdθ   U.L : 1 = cosθ $\Rightarrow \theta = 0°$

$$= \int_{\pi/2}^{0} \sin\left[2\tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right](-\sin\theta)d\theta$$

$$= \int_0^{\pi/2} \sin(\pi-\theta)\sin\theta d\theta = \int_0^{\pi/2} \sin^2\theta d\theta$$

$$= \frac{1}{2}\times\frac{\pi}{2} = \frac{\pi}{4}$$

14. $\int_0^3 \frac{3x+1}{x^2+9} dx =$ **[EAMCET 2003]**

1) $\text{Log}(2\sqrt{2}) + \frac{\pi}{12}$ 2) $\text{Log}(2\sqrt{2}) + \frac{\pi}{2}$ 3) $\text{Log}(2\sqrt{2}) + \frac{\pi}{6}$ 4) $\text{Log}(2\sqrt{2}) + \frac{\pi}{3}$

Ans: 1

Sol: $= \frac{3}{2}\int_0^3 \frac{2x}{x^2+9} dx + \int_0^3 \frac{1}{x^2+9} dx$

$$= \frac{3}{2}\left[\text{Log}(x^2+9)\right]_0^3 + \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{3}{2}\left[\text{Log}18 - \text{Log}9\right] + \frac{1}{3}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$= \frac{3}{2}\text{Log}2 + \frac{\pi}{12} = \text{Log}2^{3/2} + \frac{\pi}{12}$

$= \text{Log}\left(2\sqrt{2}\right) + \frac{\pi}{12}$

15. $\int_2^3 \frac{dx}{x^2 - x} =$ **[EAMCET 2002]**

1) $\log\frac{2}{3}$ 2) $\log\frac{4}{3}$ 3) $\log\frac{8}{3}$ 4) $\log\frac{1}{4}$

Ans: 2

Sol: $\int_2^3 \frac{dx}{x^2 - x} = \int_2^3 \frac{dx}{x(x-1)}$

$= \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x}\right)dx = \left[\log\left(\frac{x-1}{x}\right)\right]_{x=2}^3$

$= \log\frac{2}{3} - \log\frac{1}{2} = \log\frac{4}{3}$

16. $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx =$ **[EAMCET 2002]**

1) $\frac{3\pi}{128}$ 2) $\frac{3\pi}{256}$ 3) $\frac{3\pi}{572}$ 4) $\frac{3\pi}{64}$

Ans: 2

Sol: $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx = 2\int_0^{\pi/2} \sin^4 x \cos^6 x dx$

$= 2\left[\frac{3}{10}.\frac{1}{8}.\frac{5}{6}.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}\right] = \frac{3\pi}{256}$

17. $\int_0^{\pi/2} \sin^8 x \cos^2 x dx =$ **[EAMCET 2001]**

1) $\frac{\pi}{512}$ 2) $\frac{3\pi}{512}$ 3) $\frac{5\pi}{512}$ 4) $\frac{7\pi}{512}$

Ans: 4

Sol: $\int_0^{\pi/2} \sin^8 x \cos^2 x dx$

$= \frac{7}{10}.\frac{5}{8}.\frac{3}{6}.\frac{1}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{7\pi}{512}$

18. $\int_{-1}^{1} \left(ax^3 + bx\right)dx = 0$ for ; **[EAMCET 2001]**

1) any values of a and b　　2) a > 0 and b > 0 only

3) a > 0 and b < 0 only　　4) a < 0 and b < 0 only

Ans: 1

Sol: $ax^3 + bx$ is odd function

$\therefore \int_{-1}^{1} \left(ax^3 + bx\right) dx = 0$

for any values of 'a' and 'b'

19. $\left(\sum_{n=1}^{10} \int_{2n-1}^{-2n} \sin^{27} x dx\right) + \left(\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx\right)$ **[EAMCET 2000]**

1) 27^2　　2) – 54　　3) 54　　4) 0

Ans: 4

Sol: $\sin^{27} x$ is odd function

20. $\int_0^1 \frac{x}{(1-x)^{5/4}} dx =$ **[EAMCET 2000]**

1) $\frac{16}{3}$　　2) $\frac{3}{16}$　　3) $\frac{-3}{16}$　　4) $\frac{-16}{3}$

Ans: 4

Sol: $\int_0^1 \frac{x}{(1-x)^{5/4}} dx = \int_0^1 \frac{1-x}{\left[(1-1-x)^{5/4}\right]} dx \Rightarrow \int_0^1 \frac{1-x}{x^{5/4}} dx$

$= \int_0^1 \left(\frac{1}{x^{5/4}} - \frac{1}{x^{1/4}}\right) dx = \frac{-16}{3}$

21. If f(x) is integrable on [0, a] then $\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx =$ **[EAMCET 2000]**

1) 0　　2) 1　　3) a　　4) a/2

Ans: 4

Sol: $I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx$ ………………(1)

$\Rightarrow I = \int_0^a \frac{f(a-x)}{f(a-x) + f(x)} dx$ …………..(2)

(1) + (2)

$\Rightarrow 2I = \int_0^a I\, dx \Rightarrow I = \frac{a}{2}$

22. $\lim\limits_{n\to\infty}\left[\dfrac{1}{2n+1}+\dfrac{1}{2n+2}+.....+\dfrac{1}{2n+n}\right]$ **[EAMCET 2000]**

1) $\log_e\left(\dfrac{1}{3}\right)$ 2) $\log_e\left(\dfrac{2}{3}\right)$ 3) $\log_e\left(\dfrac{3}{2}\right)$ 4) $\log_e\left(\dfrac{4}{3}\right)$

Ans: 3

Sol: $\lim\limits_{n\to\infty}\left[\dfrac{1}{2n+1}+\dfrac{1}{2n+2}+.....+\dfrac{1}{2n+n}\right]$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{2n+r}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\left(2+\dfrac{r}{n}\right)}$$

$$=\int_0^1\frac{dx}{2+x}=\left[\log(2+x)\right]_0^1=\log_e\left(\frac{3}{2}\right)$$