5. BINOMIAL THEOREM

PREVIOUS EMACET BITS

1. The coefficient of x^{24} in the expansion of $\left(1+x^2\right)^{12}\left(1+x^{12}\right)\left(1+x^{24}\right)$ **[EAMCET 2009]**

   1) ${}^{12}C_6$ 2) ${}^{12}C_6+2$ 3) ${}^{12}C_6+4$ 4) ${}^{12}C_6+6$

   Ans: 2

Sol: $\left(1+x^2\right)^{12}\left(1+x^{12}+x^{24}+x^{36}\right)$

$=\left[{}^{12}C_0+{}^{12}C_1x^2+{}^{12}C_2x^4+......+{}^{12}C_{12}x^{24}\right]\left(1+x^{12}+x^{24}+x^{36}\right)$

Coefficient of x^{24} is ${}^{12}C_0+{}^{12}C_6+{}^{12}C_{12}={}^{12}C_6+2$

2. If x is numerically so small so that x^2 and higher powers of x can be neglected, then $\left(1+\frac{2x}{3}\right)^{3/2}(32+5x)^{-1/5}$ is approximately equal to **[EAMCET 2009]**

   1) $\frac{32+31x}{64}$ 2) $\frac{31+32x}{64}$ 3) $\frac{31-32x}{64}$ 4) $\frac{1-2x}{64}$

   Ans: 1

Sol: $\left[1+\frac{3}{2}\left(\frac{2x}{3}\right)\right]\left[32\left(1+\frac{5x}{32}\right)\right]^{-1/5}$

$=[1+x](32)^{-1/5}\left(1+\frac{5x}{32}\right)^{-1/5}$

$=\frac{1}{2}(1+x)\left[1-\frac{1}{5}\times\frac{5x}{32}\right]=\frac{1}{2}(1+x)\left(1-\frac{x}{32}\right)$

$=\frac{1}{2}\left(1+x-\frac{x}{32}\right)=\frac{32+31x}{64}$

3. If $\left(1+x+x^2+x^3\right)^5=\sum_{k=0}^{15}a_kx^k$ then $\sum_{k=0}^{7}a_{2k}=.....$ **[EAMCET 2008]**

   1) 128 2) 256 3) 512 4) 1024

   Ans: 3

Sol: $a_0+a_1x+a_2x^2+......+a_{15}x^{15}=\left(1+x+x^2+x^3\right)^5$

Put x = 1

$a_0+a_1+a_2+......+a_{15}=4^{15}$ ………..(1)

Put x = – 1

$a_0-a_1+a_2-......-a_{15}=0$ …………(2)

(1) + (2)

$$2\left(a_0 + a_2 + a_4 + ... + a_{14}\right) = 4^5$$

$$\therefore a_0 + a_2 + a_4 + ... + a_{14} = 512$$

4. If $a = \frac{5}{2!3} + \frac{5.7}{3!3^2} + \frac{5.7.9}{4!3^3} + ...$then $a^2 + 4a =$ **[EAMCET 2008]**

1) 21 2) 23 3) 25 4) 27

Ans: 2

Sol: $a = \frac{3.5}{2!.3^2} + \frac{3.5.7}{3!.3^3} + ...$

$2 + a = 1 + \frac{3}{3} + \frac{3.5}{3.6} + \frac{3.5.7}{3.6.9} + ...$ this one comparing with

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$$

$$nx = 1, \frac{nx(nx-x)}{2} = \frac{3.5}{3.6}$$

$$\frac{1(1-x)}{2} = \frac{3.5}{3.6} \Rightarrow 1 - x - \frac{5}{3} \Rightarrow x = \frac{-2}{3}$$

$$nx = 1 \Rightarrow n\left(-\frac{2}{3}\right) = 1 \Rightarrow n = -\frac{3}{2}$$

$$\therefore \ 2 + a = (1+x)^n = \left(1 - \frac{2}{3}\right)^{-3/2} \Rightarrow \left(\frac{1}{3}\right)^{-3/2}$$

$$2 + a = 3^{3/2} \Rightarrow (2+a)^2 = 3^3 \Rightarrow a^2 + 4a = 23$$

5. If a_k is the coefficient of x^K in the expansion of $\left(1 + x + x^2\right)^n$ for k = 0, 1, 2, ……2n, then $a_1 + 2a_2 + 3a_3 + ... + 2na_{2n}$ is equal to **[EAMCET 2007]**

1) $-a_0$ 2) 3^n 3) $n.3^{n+1}$ 4) $n.3^n$

Ans: 4

Sol: We have $\left(1 + x + x^2\right)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ...a_{2n}x^{2n}$ on differentiating both sides, we get

$$n\left(1 + x + x^2\right)^{n-1}(1 + 2x) = a_1 + 2a_2x + 3a_3x^2 + ... + 2na_{2n}x^{2n-1}$$

Put x = 1

$$a_1 + 2a_2 + 3a_3 + ... + 2na_{2n} = n.3^n$$

6. The sum of the series $\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} .... =$ **[EAMCET 2007]**

1) $\sqrt{\frac{3}{2}}-\frac{3}{4}$ 2) $\sqrt{\frac{2}{3}}-\frac{3}{4}$ 3) $\sqrt{\frac{3}{2}}-\frac{1}{4}$ 4) $\sqrt{\frac{2}{3}}-\frac{1}{4}$

Ans: 2

Sol: $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}.......+\frac{3}{4}-\frac{3}{4}$

$=\frac{3}{4}+\frac{3}{4.8}-\frac{3.5}{4.8.12}+.....-\frac{3}{4}$

$=1-\frac{1}{4}+\frac{1.3}{2.4.4}-\frac{1.3.5}{4.4.2.4.3}+.....-\frac{3}{4}$

$=\left[1+1\left(-\frac{1}{4}\right)+\frac{1(1+2)}{2!}\left(-\frac{1}{4}\right)^2+.....\right]-\frac{3}{4}$

$=\left(1-\frac{1}{4}\right)^{-1/2}-\frac{3}{4}=\sqrt{\frac{2}{3}}-\frac{3}{4}$

7. $1+\frac{2}{4}+\frac{2.5}{4.8}+\frac{2.5.8}{4.8.12}+....$ is equal to **[EAMCET 2006]**

1) $4^{-2/3}$ 2) $\sqrt[3]{16}$ 3) $\sqrt[3]{4}$ 4) $4^{3/2}$

Ans: 2

Sol: Let $s=1+\frac{2}{4}+\frac{2.5}{4.8}+....$ on comparing with

$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+.....$

We get , $nx=\frac{2}{4},\frac{nx(nx-x)}{2}=\frac{2.5}{4.8}$

$=\frac{\frac{2}{4}\left(\frac{2}{4}-x\right)}{2}=\frac{2.5}{4.8}\Rightarrow x=\frac{-3}{4}$

$n\left(-\frac{3}{4}\right)=\frac{2}{4}\Rightarrow n=-\frac{2}{3}$

$\therefore s=(1+x)^n=\left(1-\frac{3}{4}\right)^{-2/3}=\left(\frac{1}{4}\right)^{-2/3}=\sqrt[3]{16}$

8. The correct matching of List- I from List – II is **[EAMCET 2006]**

List – I | List – II

A) $(1-x)^{-n}$ (i) $\frac{x}{x+1}$

B) $(1+x)^{-n}$

C) If x > 1 then $1+\frac{1}{x}+\frac{1}{x^2}+.....$ is

D) If $|x|>1$ then $1-\frac{2}{x^2}+\frac{3}{x^4}-\frac{4}{x^6}+....$ is

(ii) $1-nx+\frac{n(n+1)}{2!}x^2.......\text{if } |x|<1$

(iii) $1+nx+\frac{n(n+1)}{2!}x^2+.......\text{if } |x|<1$

(iv) $\frac{x}{x-1}$

(v) $\frac{x^4}{(x^2+1)^2}$

(vi) $\frac{x^4}{(x^2-1)^2}$

	A	B	C	D		A	B	C	D
1)	i	iii	iv	v	2)	ii	iii	iv	v
3)	iii	ii	iv	v	4)	ii	iii	i	v

Ans:

Sol: We know that (i) $(1-x)^{-n}=1+nx+\frac{n(n+1)}{2!}x^2+....\text{if } |x|<1$

(ii) $(1+x)^{-n}=1-nx+\frac{n(n-1)}{2!}x^2+....\text{if } |x|<1$

(iii) $1+\frac{1}{x}+\frac{1}{x^2}+.........=\frac{1}{1-\frac{1}{x}}=\frac{x}{x-1}$

(iv) $1-\frac{2}{x^2}+\frac{3}{x^4}................=\frac{x^4}{(x^2+1)^2}$

9. If $(1+x)^{15}=a_0+a_1x+.....+a_{15}x^{15}$, then $\sum_{r=1}^{15} r\frac{a_r}{a_r-1}$ is equal to **[EAMCET 2005]**

1) 110 2) 115 3) 120 4) 135

Ans: 3

Sol: $\sum_{r=1}^{15} r.\frac{a_r}{a_{r-1}}=\sum_{r=1}^{15} r.\frac{n-(r-1)}{r}$

$=\sum_{r=1}^{15}\left[(n+1)-r\right]=\sum_{r=1}^{15}(16-r)$

=15 + 14 + …….+ 1

$=\frac{15\times 16}{2}=120$

10. If $|x| < \frac{1}{2}$, then the coefficient of x' in the expansion of $\frac{1+2x}{(1-2x)^2}$, is **[EAMCET 2005]**

1) $r2^r$ 2) $(2r-1)2^r$ 3) $r2^{2r+1}$ 4) $(2r+1)2^r$

Ans: 4

Sol: $\frac{1+2x}{(1-2x^2)} = (1+2x)(1-2x)^{-2}$

$$= (1+2x)\left[1+\frac{2}{1!}(2x)+\frac{2.3}{2!}(2x)^2+....+\frac{2.3.....r}{(r-1)!}(2x)^{r-1}+\frac{2.3.4....(r+1)(2x)^r}{r!}\right]$$

The coefficient of x^r = $2\frac{r!}{(r-1)!}.2^{r-1}+\frac{(r+1)!}{r!}2^r$

$= r.2^r + (r+1).2^r = 2^r(2r+1)$

11. The coefficient of $x^3y^4z^5$ in the expansion of $(xy+yz+zx)^6$ is **[EAMCET 2005]**

1) 70 2) 60 3) 50 4) none of these

Ans: 2

Sol: If the general term in the above expansion contains $x^3y^4z^5$ then r + t = 3, r + s = 4 and s + t = 5

Also, $r+s+t=6$

Solving these equation, we get r = 1, s = 3, t = 2

Coefficient $x^3y^4z^5 = \frac{6!}{1!3!2!} = 60$

12. The binomial coefficients which are in decreasing order are **[EAMCET 2004]**

1) ${}^{15}C_5, {}^{15}C_6, {}^{15}C_7$ 2) ${}^{15}C_{10}, {}^{15}C_9, {}^{15}C_8$ 3) ${}^{15}C_6, {}^{15}C_7, {}^{15}C_8$ 4) ${}^{15}C_7, {}^{15}C_6, {}^{15}C_5$

Ans: 4

Sol: The series of binomial coefficient is

$$\underbrace{{}^{15}C_0, {}^{15}C_1, {}^{15}C_2, ...{}^{15}C_7}_{\text{decreasing value}}, \underbrace{{}^{15}C_8, {}^{15}C_9}_{\text{greatest value}} .... \underbrace{{}^{15}C_{14}, {}^{15}C_{15}}_{\text{decreasing value}}$$

From the above discussion, we can say that decreasing series is ${}^{15}C_7, {}^{15}C_6, {}^{15}C_5$

$\therefore$ Option (4) is correct

13. If the coefficient $(2r + 1)^{th}$ term and $(r + 2)^{th}$ term in the expansion of $(1 + x)^{42}$ are equal, then r is equal to : **[EAMCET 2003]**

1) 12 2) 14 3) 16 4) 18

Ans: 2

Sol: Given that coefficient of $(2r + 1)^{th}$ term = coefficient of $(r + 2)^{th}$ term

${}^{43}C_{2r} = {}^{43}C_{r+1} \Rightarrow 43 = 2r + (r+1)$ (or) $2r = r+1$

$\Rightarrow r = 14 \,(\text{or})\, r = 1$

Thus r = 14

14. The coefficient of x^5 in the expansion of $\left(1+x^2\right)^5\left(1+x\right)^4$ is **[EAMCET 2003]**

1) 60 2) 50 3) 40 4) 56

Ans : 1

Sol: We have $\left(1+x^2\right)^5\left(1+x\right)^4 = \left[1+{}^5C_1x^2+{}^5C_2x^4+.....+\left(x^2\right)^5\right]$

$\left[1+{}^4C_1x+{}^4C_2x^2+{}^4C_3x^3+x^4\right]$

Coefficient to x^5

$= {}^5C_1.{}^4C_3+{}^5C_2.{}^4C_1 = 20+40 = 60$

15. If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270 then k is equal to **[EAMCET 2002]**

1) 1 2) 2 3) 3 4) 4

Ans: 3

Sol: General term in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is

$T_{r+1} = {}^5C_r\left(x^2\right)^{5-r}\left(\frac{k}{x}\right)^r = {}^5C_rk^rx^{10-3r}$

Let this term contains x then $10-3r = 1 \Rightarrow r = 2$ then

Coefficient $x = {}^5C_3k^3 = 10k^3$

$10k^3 = 270$

$k^3 = 27 \;\therefore k = 3$

16. The sum of the coefficients in the expansion of $\left(1+x+x^2\right)^n$ is **[EAMCET 2002]**

1) 2 2) 2^n 3) 3^n 4) 4^n

Ans: 3

Sol: we have $\left(1+x+x^2\right)^n$, put x = 1

$= \left(1+1+1\right)^n = 3^n$

17. In the expansion of $(1+x)^n$ the coefficients of p^{th} and $(p+1)^{th}$ terms are respectively p and q, then p + q is equal to **[EAMCET 2002]**

1) n 2) n + 1 3) n + 2 4) n + 3

Ans: 2

Sol: $T_P = {}^nC_{p-1} = P$

$T_{p+1} = {}^nC_p = q$

$$\therefore \frac{p}{q} = \frac{{}^nC_{p-1}}{{}^nC_p} \Rightarrow \frac{p}{q} = \frac{p}{n-p+1} \Rightarrow p+q = n+1$$

18. $1 + \frac{1}{4} + \frac{1.3}{4.8} + \frac{1.3.5}{4.8.12} + ... =$ **[EAMCET 2001]**

1) $\sqrt{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\sqrt{3}$ 4) $\frac{1}{\sqrt{3}}$

Ans: 1

Sol: This one comparing with $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$

$$nx = \frac{1}{4}, \frac{n(n-1)}{2!}x^2 = \frac{1.3}{4.8}$$

$$\frac{nx(nx-x)}{2} = \frac{1.3}{4.8} \Rightarrow \frac{\frac{1}{4}\left(\frac{1}{4}-x\right)}{2} = \frac{1.3}{4.8}$$

$$\Rightarrow \frac{1}{4} - x = \frac{3}{4} \Rightarrow x = -\frac{1}{2} = \; nx = \frac{1}{4}$$

$$n\left(-\frac{1}{2}\right) = \frac{1}{4} \Rightarrow n = -\frac{1}{2}$$

$$1 + \frac{1}{4} + \frac{1.3}{4.8} + ... = \left(1 - \frac{1}{2}\right)^{-1/2} = \sqrt{2}$$

19. The coefficient of x^4 in the expansion of $\frac{(1-3x)^2}{1-2x}$ is equal to **[EAMCET 2001]**

1) 1 2) 2 3) 3 4) 4

Ans: 4

Sol: we have $\frac{(1-3x)^2}{1-2x} = (1 + 9x^2 - 6x)(1-2x)^{-1}$

$$= (1 + 9x^2 - 6x)\left[1 + 2x + (2x)^2 + (2x)^3 + (2x)^4 + ...\right]$$

Coefficient of $x^4 = 16 + 9(4) - 6(8) = 4$

20. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ then $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ is equal to

[EAMCET 2001]

1) $2^n + n.2^{n-1}$ 2) $2^{n-1} + n.2^n$ 3) $2^n + (n+1)2^n$ 4) $2^{n-1} + (n-1)2^n$

Ans:1

Sol: $\left(\frac{n+1+1}{2}\right).2^n = (n+2).2^{n-1}$

$= n.2^{n-1} + 2^n$ (or)

Let $S = C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ …………..(1)

$S = C_n + 2C_{n-1} + 3C_{n-2} + ... + (n+1)C_0$

$S = ^{(n+1)}C_0 + ^nC_1 + ... + C_n$ ………………(2)

(1) + (2)

$\Rightarrow 2S = C_0(1+n+1) + C_1(2+n) + ... + C_n(n+1+1)$

$2S = (n+2)(C_0 + C_1 + C_2 + ... + C_n) = (n+2).2^n$

$S = (n+2).2^{n-1} = n.2^{n-1} + 2^n$

21. If $C_0, C_1, C_2, ...$ are binomial coefficient, then $C_1 + C_2 + C_3 + ... + Cr + ... + C_n$ is equal to **[EAMCET 2000]**

1) 2^n 2) 2^{n-1} 3) $2^n - 1$ 4) 2^{2n}

Ans: 3

Sol: we have $(1+x)^n = 1 + ^nC_1x + ^nC_2x^2 + ... ^nC_nx^n$

Put x = 1

$1 + ^nC_1 + ^nC_2 + ... + ^nC_n = 2^n$

$C_1 + C_2 + ... + C_n = 2^n - 1$

22. The coefficient of x^{-n} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$ **[EAMCET 2000]**

1) 0 2) 1 3) 2^n 4)

Ans: 2

Sol: $(1+x)^n\left(1+\frac{1}{x}\right) = \frac{(1+x)^{2n}}{x^n}$

$= x^{-n}\left[^{2n}C_0 + ^{2n}C_1x + ... + ^{2n}C_nx^{2n}\right]$

Coefficient of $x^{-n} = ^{2n}C_0 = 1$

23. If the coefficient of r^{th} term and $(2+1)^{th}$ term in the expansion of $(1+x)^{3n}$ are in the ratio 1 : 2 the r is equal to **[EAMCET 2000]**

1) $\frac{6}{5}(n+1)$ 2) $\frac{1}{3}(3n+1)$ 3) $\frac{1}{4}(n+2)$ 4) $\frac{1}{3}(3n+2)$

Ans: 2

Sol: coefficient of r^{th} term = $T_r = ^{3n}C_{r-1}$

Coefficient of (r+1)th term = $T_{r+1} = {}^{3n}C_r$

Given $\frac{T_r}{T_{r+1}} = \frac{1}{2} \Rightarrow \frac{{}^{3n}C_{r-1}}{{}^{3n}C_r} = \frac{1}{2} \Rightarrow \frac{r}{3n-r+1} = \frac{1}{2} \Rightarrow r = \frac{1}{3}(3n+1)$