COORDINATE GEOMETRY

LOCUS

SYNOPSIS AND FORMULAE

1. Locus: The path traced out by moving point under one or more given conditions is called its Locus .

2. Equation of Locus: The algebraic relation between x and y obtained by applying the geometrical conditions is called the equation of locus.

3. The locus of a point which is equidistant from the two points A (x_1, y_1) and B (x_2, y_2) is a straight line whose equation is

   $$2(x_1 - x_2)x + 2(y_1 - y_2)y = \left(x_1^2 + y_1^2\right) - \left(x_2^2 + y_2^2\right)$$

   This equation of the perpendicular bisector of the join the two points (x_1, y_1) and (x_2, y_2)

4. The locus of a point which is at a distance er1 from the the given point A (x_1, y_1) is a circle whose equation is

   $$(x - x_1)^2 + (y - y_1)^2 = r^2$$

5. The locus of a point ePl such that PA = KPB is

   (i) perpendicular bisector of AB if K = 1

   (ii) a circle of K $\neq$ 1

6. Let A, B be two points, then the locus of a point ePl such that PA + PB = K is

   (i) an ellipse if K > AB

   (ii) a line segment if K = AB

   (iii) empty set if K < AB

7. Let A, B be two points then the locus of a point P such that $|PA - PB|$ = K is

(i) a hyperbola if K < AB

(ii) union of two rays if K = AB

(iii) empty set if K > AB

8. If A, B are two points then the locus of a point P such that

$PA^2 + PB^2 = K\ PC^2$ is

(i) a straight line if K = 2

(ii) a circle if K $\neq$ 2 and K > 0

9. Let a, b be two points then the locus of P such that the area of triangle PAB is “ ie., a pair of parallel lines which are parallel to $\overline{AB}$ and at a distance of $\frac{2A}{AB}$ from $\overline{AB}$

10. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents

(i) a pair of lines if “ = 0 ; $h^2 > ab$

(ii) a pair of parallel lines if “ = 0 ; $h^2 = ab$

(iii) a pair of perpendicular lines if “ = 0 ; a + b = 0

11. If A = (a, b) B = (-a, b) then the locus of P such that PA + PB = k or $|PA - PB|$ = K is $\frac{4x^2}{k^2} + \frac{4(y-b)^2}{k^2 - 4a^2} = 1$

12. If A = (a, b) B = (a, -b) then the locus of P such that PA + PB = k or $|PA - PB|$ = K is

$$\frac{4(x-a)^2}{k^2 - 4b^2} + \frac{4y^2}{k^2} = 1$$

13. Let A, B be two given points. The locus of P such that the area of “PAB is k sq. units is a pair of parallel straight lines

14. Let A, B be two fixed points. The locus P such that $\angle APB = 90^0$ is a circle on the line joining A, B as the ends of a diameter

15 The ends of a rod of length K moves on two positive coordinate axes. The locus of the point on the rod, which divides it in the ratio l : m is

$$\frac{x^2}{m^2}+\frac{y^2}{l^2}=\frac{k^2}{(l+m)^2} \quad \text{or} \quad \frac{x^2}{l^2}+\frac{y^2}{m^2}=\frac{k^2}{(l+m)^2}$$

16. A st. line passing through the point (x_1, y_1) meets the positive coordinate axis at A, B. The locus of the point P which divides AB in the ratio l : m is $\frac{lx^2}{x}+\frac{my_1}{y}$ = l + m or $\frac{mx_1}{x}+\frac{ly_1}{y}$ =l + m