COORDINATE GEOMETRY

CHANGE OF AXES

SYNOPSIS AND FORMULAE

1. If origin is shifted to the point (x_1, y_1) with new axes parallel to the original axes and (X, Y) are the new co-ordinates of the point (x, y), then $x = X+x_1$, $y = Y+y_1$.

2. To remove the first degree terms from the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ shift the origin to the point

$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$ and the new transformed equation will be

$$aX^2 + 2hXY + bY^2 + \left(ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c\right) = 0.$$

3. To remove the first degree terms from the equation $ax^2 + by^2 + 2gx+2fy+c=0$ origin must be shifted to the point $\left(-\frac{g}{a}, -\frac{f}{b}\right)$.

4. To remove the first degree terms from the equation $2hxy+2gx+2fy+c=0$ origin must be shifted to the point $\left(-\frac{f}{h}, -\frac{g}{h}\right)$.

5. To remove the first degree terms from the equation $xy+ax+by+c=0$ origin must be shifted to the point (-b, -a).

6. If axes are rotated through an angle q and (X, Y) are the new co-ordinates of (x,y), then $x = X\cos q - Y\sin q$, $y = X\sin q + Y\cos q$ and

$X = x\cos q + y\sin q$, $Y = -x\sin q + y\cos q$.

	X	Y
x	cos	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

7. If on changing the axes, the expression $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ transforms to $AX^2+2HXY+BY^2+2GX+2FY+C$, then $A + B = a + b$,

$AB - H^2 = ab - h^2$ under transformation of axes $a + b$,

$ab - h^2$ are called the invariants.

8. On rotating the axes, the distance from origin to the line does not change.

9. On translating the axes from one origin to another, slope will not change.

10. The angle of rotation of the axes so that the equation $ax+by+c= 0$ may be reduced to $X = K$ is $\quad Tan^{-1}(b/a)$

11. The angle of rotation of the axes so that the equation ax+by+c= 0 may be reduced to Y=K is $\mathrm{Tan}^{-1}(-a/b)$.

12. The xy term is removed from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, by rotation of axes through an angle $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$

13. (a) The point to which the origin has to be shifted to eliminate first degree terms (x, y terms) in

$f = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is obtained by solving

$$\frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0$$

(b) The first degree terms are removed from the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by

translation of axes to the point $\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}$. In this case, the transformed equation is $aX^2 + 2hXY + bY^2 + (gx_1 + fy_1 + c) = 0$

14. H to remove the first degree terms from $ax^2 + by^2 + 2gx + 2fy + c = 0$, the origin is shifted to the point $\left(\frac{-g}{a}, \frac{-f}{b}\right)$

In this case, the transformed equation is a

$$X^2 + bY^2 + \left(\frac{-g}{a} - \frac{f}{b} + c\right) = 0$$

H to remove the first degree terms from

$2hxy + 2gx + 2fy + c = 0$, the origin is shifted to the point

$\left(\frac{-f}{h}, \frac{-g}{h}\right)$ In this case, the transformed equation is $2hXY + c = 0$

15. The condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, to take the form $aX^2 + 2hXY + bY^2 = 0$, when the axes are translated is $abc + 2fgh - af^2 - bg^2 - Ch^2 = 0$.

16. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ has transformed to $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$, when the origin is shifted to (l, m)

then A = a; B = b; H= h; $2G = \left(\frac{\partial f}{\partial x}\right)_{(l,m)}$ $2F = \left(\frac{\partial f}{\partial y}\right)_{(l,m)}$ C = f (l, m)