TRIPLE PRODUCT AND PRODUCT OF FOUR VECTORS

PREVIOUS EAMCET BITS

1. The volume of the tetrahedron having the edges $\vec{i}+2\vec{j}-\vec{k}, \vec{i}+\vec{j}+\vec{k}, \vec{i}-\vec{j}+\lambda\vec{k}$ as conterminous, is 2/3 cubic units. Then λ **[EAMCET 2009]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol. $V=\frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{2}{3}$ cubc units

$\Rightarrow \lambda=1$

2. If a = i + j + k , b = i – j + k, c = i + j + k, d = i – j – k, then observe the following lists **[EAMCET 2008]**

List – I	List – II
i) $\bar{a}.\bar{b}$	A) $\bar{a}.\bar{d}$
ii) $\bar{b}.\bar{c}$	B) 3
iii) $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$	C) $\bar{b}.\bar{d}$
iv) $b\times c$	D) $2\bar{j}-2\bar{k}$
	E) $2\bar{j}+2\bar{k}$
	F) 4

The correct match of List-I to List – II

	i	ii	iii	iv		i	ii	iii	iv
1)	C	A	B	F	2)	C	A	F	E
3)	A	C	B	F	4)	A	C	F	D

Ans: 2

Sol. $a.b=\left(i+j+k\right).\left(i-j+k\right)=1-1+1=1$

$b.c=\left(i-j+k\right).\left(i+j-k\right)=1-1-1=-1$

$$\left[abc\right]=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=1\left(1-1\right)-1\left(-1-1\right)+1\left(1+1\right)=0+2+2=4$$

$$b\times c=\begin{vmatrix}i & j & k\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=i\left(1-1\right)-j\left(-1-1\right)+k\left(1+1\right)=2j+2k$$

$a.d=\left(i+j+k\right).\left(i-j-k\right)=1-1-1=-1$

$b.d=\left(i-j+k\right).\left(i-j-k\right)=1+1-1=-1$

3. Let a be a unit vector, b = 2i + j – k and c= i + 3k, the maximum value of [a b c] is **[EAMCET 2008]**

1) – 1 2) $\sqrt{10}+\sqrt{6}$ 3) $\sqrt{10}-\sqrt{6}$ 4) $\sqrt{59}$

Ans: 4

Sol. $b\times c=\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix}=i(3-0)-j(6+1)+k(0-1)=3i-7j-k$

$[abc]=a.(b\times c)=a.(3i-7j-k)$

$=|a||3i-7j-k|\cos\theta$ where $\theta=(a,3i-7j-k)$

$=\sqrt{9+49+1}.\cos\theta$

$=\sqrt{59}\cos\theta$

$\therefore$ Maximum value of $\left[\bar{a}\bar{b}\bar{c}\right]$ is $\sqrt{59}$

4. The volume (in cubic units) of the tetrahedron with edges $\vec{i}+\vec{j}+\vec{k}, \vec{i}-\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}-\vec{k}$ is **[EAMCET 2007]**

1) 4 2) 2/3 3) 1/6 4) 1/3

Ans: 2

Sol. $V=\frac{1}{6}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}=\frac{2}{3}$

5. $\vec{i}-2\vec{j}, 3\vec{j}+\vec{k}$ and $\lambda\vec{i}-3\vec{j}$ are coplanar then = **[EAMCET 2006]**

1) – 1 2) 1/2 3) –3/2 4) 2

Ans: 3

Sol. $\bar{a},\bar{b},\bar{c}$ are coplanar $\Rightarrow\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0\Rightarrow\begin{vmatrix} 1 & -2 & 0 \\ 0 & 3 & 1 \\ \lambda & 3 & 0 \end{vmatrix}=0$

$1(0-3)+2(0-\lambda)+0(0-3\lambda)=0$

$\lambda=\frac{-3}{2}$

6. If the volume of the parallelopiped with coterminous edges **[EAMCET 2006]**
$4\vec{i}+5\vec{j}+\vec{k}, -\vec{j}+\vec{k}$ and $3\vec{i}+9\vec{j}+p\vec{k}$ is 34 cubic units, then p = ……..

1) 4 2) –13 3) 13 4) 6

Ans: 1 or 3

Sol. Volume = |[a b c]|= $\begin{vmatrix} 4 & 5 & 1 \\ 0 & -1 & 1 \\ 3 & 9 & p \end{vmatrix}=34$

$\Rightarrow|4p+18|=34\Rightarrow p=-13$ or 4

7. Observe the following lists **[EAMCET 2005]**

List – I	List – II
A) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$	1) $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\left(\vec{a}\vec{b}\right)$
B) $(\vec{c}\times\vec{a})\times\vec{b}$	2) $\left(\vec{a}.\vec{b}\right)\vec{b}-\left(\vec{a}\vec{b}\right)\vec{c}$
C) $\vec{a}\times\left(\vec{b}\times\vec{b}\right)$	3) $\vec{a}.\vec{b}\times\vec{c}$

D) $\vec{a}.\vec{b}$ 4) $|\vec{a}||\vec{b}|$

5) $(\vec{b}.\vec{c})\vec{a}-(\vec{a}.\vec{b})\vec{c}$

	A	B	C	D		A	B	C	D
1)	1	2	3	4	2)	3	5	2	1
3)	3	2	5	1	4)	2	3	4	1

Ans: 2

Sol. $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=\bar{a}.\left(\bar{b}\times\bar{c}\right)$

$(\bar{c}\times\bar{a})\times\bar{b}=(\bar{b}.\bar{c})\bar{a}-(\bar{a}.\bar{b})\bar{c}$

$\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}$

$\bar{a}.\bar{b}=|a||b|\cos(\bar{a}.\bar{b})$

8. $\bar{c}.(\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})=$ **[EAMCET 2004]**

1) $\bar{c}.\bar{b}\times\bar{a}$ 2) $\vec{0}$ 3) $\bar{c}.\bar{a}\times\bar{b}$ 4) $\bar{a}.\bar{c}\times\bar{b}$

Ans: 1

Sol. $(\bar{c}.\bar{b}+\bar{c}.\bar{c})\times(\bar{a}+\bar{b}+\bar{c})=\bar{c}.(\bar{b}\times\bar{a})=\left[\bar{c}\ \bar{b}\ \ \bar{a}\right]$

9. If $3\vec{i}+3\vec{j}+\sqrt{3}\vec{k},\vec{i}+\vec{k},\sqrt{3}\vec{i}+\sqrt{3}\vec{j}+\lambda\vec{k}$ are coplanar, then $\lambda=$ **[EAMCET 2004]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol. $\begin{vmatrix}3 & 3 & \sqrt{3}\\ 1 & 0 & 1\\ \sqrt{3} & \sqrt{3} & \lambda\end{vmatrix}=0\Rightarrow\lambda=1$

10. If $\bar{a}=\vec{i}+\vec{j}+\vec{k},\bar{b}=\vec{i}+\vec{j},c=\vec{i}$ and $(\bar{a}\times\bar{b})\times\bar{c}=\lambda\bar{a}+\mu\bar{b}$, then $\lambda+\mu$.... **[EAMCET 2003]**

1) 0 2) 1 3) 2 4) 3

Ans: 1

Sol. $(\bar{a}\times\bar{b})\times\bar{c}=(\bar{a}.\bar{c})\bar{b}-(\bar{b}.\bar{c})\bar{a}\ =\ \bar{b}-\bar{a}$

$\lambda=-1;\mu=1\Rightarrow\lambda+\mu=0$

11. If $\left[\vec{a}\,\vec{b}\ \vec{c}\right]$=3, then the volume (in cubic units) of the parallelopiped with $2\bar{a}+\vec{b},2\vec{b}+\bar{c}$ and $2\bar{c}+\bar{a}$ as coterminous edges is **[EAMCET 2002]**

1) 15 2) 22 3) 25 4) 27

Ans: 4

Sol. $=\left[2\vec{a}+\vec{b}\ \ 2\vec{b}+\vec{c}\ \ 2\vec{c}+\vec{a}\right]=\begin{vmatrix}2 & 1 & 0\\ 0 & 2 & 1\\ 1 & 0 & 2\end{vmatrix}\left[\bar{a}\bar{b}\bar{c}\right]=9.3=27$

12. $(\vec{a}+\vec{b}).(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})=$ **[EAMCET 2002]**

1) 0 2) $-\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ 3) $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ 4) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

Ans:

Sol. $\left(\vec{a}+\vec{b}\right).\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\begin{vmatrix}1&1&0\\0&1&1\\1&1&1\end{vmatrix}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$=\left[0-1(-1)+0\right]\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

13. $\left[\bar{i}-\bar{j}\quad \bar{j}-\bar{k}\quad \bar{k}-\bar{i}\right]=$ **[EAMCET 2001]**

1) 0 2) 1 3) 3 4) 2

Ans: 1

Sol. $\begin{vmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{vmatrix}=0$

14. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=$ **[EAMCET 2001]**

1) 1 2) $\vec{a}$ 3) $\vec{b}$ 4) $\vec{O}$

Ans: 4

Sol. $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar

$\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$ are parallel

$\therefore\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\vec{O}$

15. If $\vec{a}=2\vec{i}+3\vec{j}-4\vec{k}$, $\vec{b}=\vec{i}+\vec{j}+\vec{k}$ and $\vec{c}=4\vec{i}+2\vec{j}+3\vec{k}$ and $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|=$ **[EAMCET 2000]**

1) $\sqrt{10}$ 2) 1 3) 2 4) $\sqrt{5}$

Ans: 4

Sol. $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|=\left|\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}\right|$

$=\left|-2i-k\right|=\sqrt{4+1}=\sqrt{5}$

16. $\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)=$ **[EAMCET 2000]**

1) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$ 2) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{b}$ 3) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{a}$ 4) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$

Ans: 1

Sol. $\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)=\left(\left(\vec{b}\times\vec{c}\right)\vec{a}\right)\vec{c}-\left(\left(\vec{b}\times\vec{c}\right)\vec{c}\right)\vec{a}$

$\Rightarrow\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}-0=\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$

❋❋❋❋