

## TRANSFORMATIONS

### PREVIOUS EAMCET BITS

1.  $\frac{\cos x}{\cos(x-2y)} = \lambda \Rightarrow \tan(x-y) \tan y =$  [EAMCET 2009]

- 1)  $\frac{1+\lambda}{1-\lambda}$       2)  $\frac{1-\lambda}{1+\lambda}$       3)  $\frac{\lambda}{1+\lambda}$       4)  $\frac{\lambda}{1-\lambda}$

Ans: 2

Sol.  $\frac{\cos(x-2y)}{\cos x} = \frac{1}{\lambda}$ , using component to and dividendo:

$$\tan(x-y) \tan y = \frac{1-\lambda}{1+\lambda}$$

2.  $\cos A \cos 2A \cos 4A \dots \cos 2^{n-1} A =$  [EAMCET 2009]

- 1)  $\frac{\sin 2^n A}{2^n \sin A}$       2)  $\frac{2^n \sin 2^n A}{\sin A}$       3)  $\frac{2^n \sin A}{\sin 2^n A}$

Ans: 1

Sol.  $\frac{1}{2 \sin A} [2 \sin A \cos A \cos 2A \cos 4A \dots]$

$$\cos[(2^{n-1} A)] = \frac{\sin(2^n A)}{2^n \sin A}$$

3. If  $\alpha + \beta + \gamma = 2\theta$ , then  $\cos \theta + \cos(\theta - \alpha) + \cos(\theta - \beta) + \cos(\theta - \gamma) =$  [EAMCET 2008]

- 1)  $4 \sin \frac{\alpha}{2} \cos \frac{\beta}{2} \sin \frac{\gamma}{2}$       2)  $4 \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2}$

- 3)  $4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$       4)  $4 \sin \alpha \sin \beta \sin \gamma$

Ans: 2

Sol.  $\cos \theta + \cos(\theta - \alpha) + \cos(\theta - \beta) + \cos(\theta - \gamma)$

$$= 2 \cos\left(\frac{2\theta - \alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right) + 2 \cos\left(\frac{2\theta - \beta - \gamma}{2}\right) \cos\left(\frac{\beta - \gamma}{2}\right)$$

$$= 2 \cos\left(\frac{\beta + \gamma}{2}\right) \cos\left(\frac{\alpha}{2}\right) + 2 \cos\left(\frac{\alpha}{2}\right) \cos\left(\frac{\beta - \gamma}{2}\right)$$

$$= 2 \cos\left(\frac{\alpha}{2}\right) \left[ \cos\left(\frac{\beta + \gamma}{2}\right) + \cos\left(\frac{\beta - \gamma}{2}\right) \right] = 4 \cos\left(\frac{\alpha}{2}\right) \cos\left(\frac{\beta}{2}\right) \cos\left(\frac{\gamma}{2}\right)$$

4.  $\sin A + \sin B = \sqrt{3}(\cos B - \cos A) \Rightarrow \sin 3A + \sin 3B =$  [EAMCET 2007]

- 1) 0      2) 2      3) 1      4) -1

Ans: 1

Sol. Given  $\sin A + \sin B = \sqrt{3}(\cos B - \cos A)$

$$\left(\frac{\sqrt{3}}{2}\right)\cos A + \sin A\left(\frac{1}{2}\right) = \cos B\left(\frac{\sqrt{3}}{2}\right) - \sin B\left(\frac{1}{2}\right)$$

$$\Rightarrow \cos\left(A - \frac{\pi}{6}\right) = \cos\left(B + \frac{\pi}{6}\right)$$

$$A = -B \Rightarrow \sin 3A + \sin 3B = 0$$

5.  $A + C = 2B \Rightarrow \frac{\cos C - \cos A}{\sin A - \sin C}$

[EAMCET 2005]

- 1)  $\cot B$       2)  $\cot 2B$

- 3)  $\tan 2B$

- 4)  $\tan B$

Ans: 4

Sol.  $\frac{A+C}{2} = B$

$$\frac{\cos C - \cos A}{\sin A - \sin C} = \frac{2 \sin \frac{A+C}{2} \sin \frac{A-C}{2}}{2 \cos \frac{A+C}{2} \sin \frac{A-C}{2}} = \tan B$$

6.  $A + B = C \Rightarrow \cos^2 A + \cos^2 B + \cos^2 C - 2 \cos A \cos B \cos C =$

- 1) 1      2) 2

- 3) 0

- 4) 3

Ans: 1

Sol.  $A + B = C$

$$\begin{aligned} \cos^2 A + \cos^2 B + \cos^2 C &= 1 + \cos(A+B)\cos(A-B) + \cos^2 C \\ &= 1 + \cos C [\cos(A-B) + \cos(A+B)] \\ &= 1 + \cos(2 \cos A \cos B) \end{aligned}$$

$$\cos^2 A + \cos^2 B + \cos^2 C - 2 \cos A \cos B \cos C = 1$$

7. In  $\Delta ABC$ ,  $\cos\left(\frac{B+2C+3A}{2}\right) + \cos\left(\frac{A-B}{2}\right) =$

- 1) -1      2) 0      3) 1

- 4) 2

Ans: 2

Sol.  $\cos\left(\frac{\pi+C+2A}{2}\right) + \cos\left(\frac{A-B}{2}\right)$

$$= \cos\left(\frac{2\pi-B+A}{2}\right) + \cos\left(\frac{A-B}{2}\right) = 0$$

8.  $\cos \alpha \sin(\beta - \gamma) + \cos \beta \sin(\gamma - \alpha) + \cos \gamma \sin(\alpha - \beta) = \dots$

[EAMCET 2003]

- 1) 0      2) 1/2      3) 1

- 4)  $4 \cos \alpha \cos \beta \cos \gamma$

Ans: 1

Sol.  $\Sigma \cos \alpha \sin(\beta - \gamma)$

$$= \Sigma \cos \alpha (\sin \beta \cos \gamma - \cos \beta \sin \gamma) = 0$$

9.  $\sin 47^\circ - \sin 25^\circ + \sin 61^\circ - \sin 11^\circ =$

[EAMCET 2003]

- 1)  $\cos 7^\circ$       2)  $\sin 7^\circ$

- 3)  $2 \cos 7^\circ$

- 4)  $2 \sin 7^\circ$

Ans: 1

Sol.  $\sin 47^\circ - \sin 25^\circ + \sin 61^\circ - \sin 11^\circ$

$$\begin{aligned}
 &= \sin 47^\circ + \sin 61^\circ - (\sin 25^\circ + \sin 11^\circ) \\
 &= 2 \sin 54^\circ \cos 7^\circ - 2 \sin 18^\circ \cos 7^\circ \\
 &= 2 \cos 7^\circ \left( \frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4} \right) = \cos 7^\circ
 \end{aligned}$$

10. If  $A + B + C = 270^\circ$ , then  $\cos 2A + \cos 2B + \cos 2C + 4\sin A \sin B \sin C = \dots$  [EAMCET 2003]
- 1) 0      2) 1      3) 2      4) 3  
Ans: 2

Sol.  $\cos 2A + \cos 2B + \cos 2C = 1 - 4 \sin A \sin B \sin C$   
(or) Put  $A = B = C = 90^\circ$

11.  $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ =$  [EAMCET 2002]
- 1)  $\frac{1}{2}$       2) 0      3)  $-\frac{1}{4}$       4)  $\frac{3}{4}$   
Ans: 4

$$\begin{aligned}
 &\text{Sol. } \cos^2 76^\circ + 1 - \sin^2 16^\circ - \frac{1}{2}(2 \cos 76^\circ \cos 16^\circ) \\
 &= 1 + \cos(76^\circ + 16^\circ) \cos(76^\circ - 16^\circ) - \frac{1}{2}(\cos 92^\circ + \cos 16^\circ) \\
 &= 1 + \frac{1}{2} \cos 92^\circ - \frac{1}{2} \cos 92^\circ - \frac{1}{4} = \frac{3}{4} \\
 &= \frac{3}{4}
 \end{aligned}$$

12.  $\sum_{k=1}^3 \cos^2 \left( (2k-1) \frac{\pi}{12} \right) =$
- 1) 0      2)  $\frac{1}{2}$       3)  $-\frac{1}{2}$       4)  $\frac{3}{2}$   
Ans: 4

$$\begin{aligned}
 &\text{Sol. } \cos^2 \frac{\pi}{12} + \cos^2 \frac{3\pi}{12} + \cos^2 \frac{5\pi}{12} \\
 &= \frac{1 + \cos 30^\circ}{2} + \frac{1 + \cos 90^\circ}{2} + \frac{1 + \cos 150^\circ}{2} = \frac{3}{2}
 \end{aligned}$$

13. If  $\operatorname{cosec} \theta = \frac{p+q}{p-q}$ , then  $\cot \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$  [EAMCET 2001]
- 1)  $\sqrt{\frac{p}{q}}$       2)  $\sqrt{\frac{q}{p}}$       3)  $\sqrt{pq}$       4)  $pq$   
Ans: 2

$$\begin{aligned}
 &\text{Sol. } \frac{1}{\sin \theta} = \frac{p+q}{p-q} \Rightarrow \frac{1+\sin \theta}{1-\sin \theta} = \frac{p}{q} \\
 &\Rightarrow \frac{1-\cos \left( \frac{\pi}{2} + \theta \right)}{1+\cos \left( \frac{\pi}{2} + \theta \right)} = \frac{p}{q} \Rightarrow \frac{2 \sin^2 \left( \frac{\pi}{4} + \frac{\theta}{2} \right)}{2 \cos^2 \left( \frac{\pi}{4} + \frac{\theta}{2} \right)} = \frac{p}{q}
 \end{aligned}$$

$$\therefore \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\frac{p}{p}}$$

14. If  $\sin \alpha + \sin \beta = a, \cos \alpha + \cos \beta = b$ , then  $\sin(\alpha + \beta) =$  [EAMCET 2000]

- 1)  $ab$       2)  $a + b$       3)  $\frac{2ab}{a^2 - b^2}$       4)  $\frac{2ab}{a^2 + b^2}$

Ans: 4

- Sol.  $\sin \alpha + \sin \beta = a; \cos \alpha + \cos \beta = b$

$$2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) = a; 2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) = b$$

$$\therefore \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}$$

$$\sin(\alpha + \beta) = \frac{2 \tan\left(\frac{\alpha+\beta}{2}\right)}{1 + \tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2ab}{a^2 + b^2}$$

15. If  $\tan \theta_1 = k \cot \theta_2$ , then  $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} =$  [EAMCET 2000]

- 1)  $\frac{1+k}{1-k}$       2)  $\frac{1-k}{1+k}$       3)  $\frac{k+1}{k-1}$

Ans: 1

$$\begin{aligned} \frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} &= \frac{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2} \\ \Rightarrow \frac{1 + \tan \theta_1 \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2} &= \frac{1+k}{1-k} \end{aligned}$$

- 4)  $\frac{k-1}{k+1}$

