SCALAR (DOT) PRODUCT OF TWO VECTORS

PREVIOUS EAMCET BITS

1. If m_1, m_2 , m_3 and m_4 are respectively the magnitudes of the vectors **[EAMCET 2009]**

   $\vec{a}_1 = 2\vec{i} - \vec{j} + \vec{k}, \vec{a}_2 = 3\vec{i} - 4\vec{j} - 4\vec{k}$, $\vec{a}_3 = \vec{i} - \vec{j} + \vec{k}$ and $\vec{a}_4 = -\vec{i} + 3\vec{j} + \vec{k}$

   Then the correct order of m_1, m_2, m_3, m_4 is

   1) $m_3 < m_1 < m_4 < m_2$ 2) $m_3 < m_1 < m_2 < m_4$

   3) $m_3 < m_4 < m_1 < m_2$ 4) $m_3 < m_4 < m_2 < m_1$

   Ans: 1

Sol. $m_1 = \sqrt{6}, m_2 = \sqrt{41}, m_3 = \sqrt{3}, m_4 = \sqrt{11}$

$\therefore\ m_3 < m_1 < m_4 < m_2$

2. Suppose $\vec{a} = \lambda\vec{i} - 7\vec{j} + 3\vec{k}, b = \lambda\vec{i} + \vec{j} + 2\lambda\vec{k}$. If the angle between $\vec{a}$ and $\vec{b}$ is greater than 90°, then λ satisfies the inequality : **[EAMCET 2009]**

   1) $-7 < \lambda < 1$ 2) $\lambda > 1$ 3) $1 < \lambda < 7$ 4) $-5 < \lambda < 1$

   Ans: 1

Sol. $\bar{a}.\bar{b} < 0$

$\Rightarrow \lambda^2 + 6\lambda - 7 < 0$

$(\lambda - 1)(\lambda + 7) < 0$

$-7 < \lambda < 1$

3. If the position vectors of A, B and C are respectively $2i - j + k, i - 3j - 5k$ and $3i - 4j - 4k$, then $\cos^2 A$ = **[EAMCET 2008]**

   1) 0 2) 6/41 3) 35/41 4) 1

   Ans: 3

Sol. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (i - 3j - 5k) - (2i - j + k) = -i - 2j - 6k$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (3i - 4j - 4k) - (2i - j + k) = i - 3j - 5k$

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{(-i - 2j - 6k)(i - 3j - 5k)}{|-i - 2j - 6k||i - 3j - 5k|}$$

$= \frac{-1+6+30}{\sqrt{1+4+36}\sqrt{1+9+25}} \frac{35}{\sqrt{35}\sqrt{41}} = \sqrt{\frac{35}{41}}$

$\therefore \cos^2 A = \frac{35}{41}$

4. If $\vec{a} = \vec{i} - \vec{j} - \vec{k}$ and $\vec{b} = \lambda\vec{i} - 3\vec{j} + \vec{k}$ and the orthogonal projection of $\vec{b}$ on $\vec{a}$ is $\frac{4}{3}\left(\vec{i} - \vec{j} - \vec{k}\right)$,

then λ = **[EAMCET 2007]**

1) 0 2) 2 3) 12 4) –1

Ans: 2

Sol. Orthogonal projection of b on a $= \frac{(b.a)a}{|a|^2}$

$\left(\frac{\lambda+3-1}{3}\right)(i - J - k) = \frac{4}{3}(i - J - k) \Rightarrow \lambda = 2$

5. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = \sqrt{37}$, then the angle between $\vec{a}$ and $\vec{b}$ is

[EAMCET 2006]

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

Ans: 4

Sol. $\bar{a} + \bar{b} = -\bar{c}$ Squaring o.b.s

$|\bar{a}|^2 + |\bar{b}|^2 + 2|a||b|\cos(\bar{a}, \bar{b}) = |\bar{c}|^2$

$9 + 16 + 24\cos(\bar{a}, \bar{b}) = 37$

$\cos(\bar{a}, \bar{b}) = \frac{1}{2} \Rightarrow (\bar{a}, \bar{b}) = \frac{\pi}{3}$

6. $\bar{a}.\bar{k} = \bar{a}.(2\bar{i} + \bar{j}) = \bar{a}(\bar{i} + \bar{j} + 3\bar{k}) = 1 \Rightarrow \bar{a}$ **[EAMCET 2006]**

1) $\bar{i} - \bar{k}$ 2) $\frac{1}{3}(3\bar{i} + \bar{j} - 3\bar{k})$ 3) $\frac{1}{3}(\bar{i} + \bar{j} + \bar{k})$ 4) $\frac{1}{3}(3\bar{i} - 3\bar{j} + \bar{k})$

Ans: 4

Sol. Let $\bar{a} = a_1 i + a_2 j + a_3 k$

$\bar{a}.i = 1 \Rightarrow a_1 = 1$

$\bar{a}.(2i + j) = 1 \Rightarrow 2a_1 + a_2 = 1 \Rightarrow a_2 = 1 - 2 = -1$

$\bar{a}.(i+j+3k)=1 \Rightarrow a_1+a_2+3a_3=1$

$\Rightarrow 3a_3=1 \Rightarrow a_3=\frac{1}{3}$

$\therefore \bar{a}=\frac{1}{3}[3i-3j+k]$

7. If the vector $\vec{a}=2\vec{i}+3\vec{j}+6\vec{k}$ and $\vec{b}$ are collinear and $|\vec{b}|=21$, then $\vec{b}=$ **[EAMCET 2005]**

1) $\pm(2\vec{i}+3\vec{j}+6\vec{k})$ 2) $\pm 3(2\vec{i}+3\vec{j}+6\vec{k})$ 3) $(\vec{i}+\vec{j}+\vec{k})$ 4) $\pm 21(2\vec{i}+3\vec{j}+6\vec{k})$

Ans: 2

Sol. $\bar{a}=t(\bar{b})$

$|\bar{a}|=|t||\bar{b}| \Rightarrow |t|=\frac{7}{21}=\frac{1}{3}$

$t=\pm\frac{1}{3}$ $\therefore \bar{b}=\pm 3(\bar{a})$

8. If the vectors $\vec{i}+3\vec{j}+4\vec{k}, \lambda\vec{i}-4\vec{j}+\vec{k}$ are orthogonal to each other, then $\lambda=$ **[EAMCET 2004]**

1) 5 2) – 5 3) 8 4) – 8

Ans: 3

Sol. $\vec{a}.\vec{b}=0 \Rightarrow \lambda-12+4=0 \Rightarrow \lambda=8$

9. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}=\vec{b}+\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{2}$: here $a=|a|, b=|b|, c=|c|$ **[EAMCET 2003]**

1) $a^2=b^2+c^2$ 2) $b^2=c^2+a^2$ 3) $c^2=a^2+b^2$ 4) $2a^2-b^2=c^2$

Ans: 1

Sol. $a^2=(b+c)^2$

$a^2=b^2+c^2+2(b.c)$

$\Rightarrow a^2=b^2+c^2 \left(\because (b.c)=\pi/2\right)$

10. If $\vec{a}.\vec{i}=\vec{a}.(\vec{i}+\vec{j})=\vec{a}.(\vec{i}+\vec{j}+\vec{k})$ then $\vec{a}=$ **[EAMCET 2002]**

1) $\vec{i}$ 2) $\vec{j}$ 3) $\vec{k}$ 4) $\vec{i}+\vec{j}+\vec{k}$

Ans: 1

Sol. By verification $\vec{a}=i$

11. The orthogonal projection of $\vec{a}$ on $\vec{b}$ is **[EAMCET 2002]**

1) $\frac{(\vec{a}.\vec{b})\vec{a}}{|\vec{a}|^2}$ 2) $\frac{(\vec{a}.\vec{b})\vec{b}}{|\vec{b}|^2}$ 3) $\frac{\vec{a}}{|\vec{a}|}$ 4) $\frac{\vec{b}}{|\vec{a}|}$

Ans: 2

Sol. $(\vec{a}.\vec{b})\frac{\vec{b}}{|\vec{b}|^2}$

12. If θ is an acute angle and the vector $(\sin\theta)\vec{i}+(\cos\theta)\vec{j}$ is perpendicular to the vector $\vec{i}-\sqrt{3}\vec{j}$ then θ = **[EAMCET 2000]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{5}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

Ans: 4

Sol. The given vectors are $\perp$ er then $(\sin\theta i+\cos\theta j).(i-\sqrt{3}j)=0$

$\sin\theta-\sqrt{3}\cos\theta=0 \Rightarrow \tan\theta=\sqrt{3}$

$\sin\theta-\sqrt{3}\cos\theta=0 \Rightarrow \tan\theta=\sqrt{3} \Rightarrow \theta=\frac{\pi}{3}$

13. If two out of the three vector $\vec{a}+\vec{b}+\vec{c}$ are unit vectors $\vec{a}+\vec{b}+\vec{c}=0$ and $2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{a}.\vec{c})+3=0$, then the third vector is of length **[EAMCET 2000]**

1) 3 2) 2 3) 1 4) 0

Ans: 3

Sol. $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow (\vec{a}+\vec{b}+\vec{c})^2=0$

$\therefore\ a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$1+1+c^2-3=0 \Rightarrow c^2=1$

$\therefore\ |\vec{c}|=1$

✹✹✹