PROPERTIES OF TRIANGLES

PREVIOUS EAMCET BITS

1. In any ΔABC, $a(b\cos C - c\cos B) =$ **[EAMCET 2009]**

1) $b^2 + c^2$ 2) $b^2 - c^2$ 3) $\frac{1}{b} + \frac{1}{c}$ 4) $\frac{1}{b^2} - \frac{1}{c^2}$

Ans: 2

Sol. $\frac{1}{2}(2ab\cos C - 2ca\cos B)$

$\frac{1}{2}\left[a^2 + b^2 - c^2 - c^2 - a^2 + b^2\right]$

$= b^2 - c^2$

2. In a ΔABC $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ **[EAMCET 2009]**

1) $\cos^2 A$ 2) $\cos^2 B$ 3) $\sin^2 A$ 4) $\sin^2 B$

Ans: 3

Sol. $\frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$

$= \frac{4\Delta^2}{b^2c^2} = \left(\frac{2\Delta}{bc}\right)^2 = \sin^2 A$

3. In ΔABC if $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$ then C = **[EAMCET 2008]**

1) 90° 2) 60° 3) 45° 4) 30°

Ans: 2

Sol. Given $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$

$\Rightarrow 1 + \frac{b}{a+c} + 1 + \frac{a}{a+c} = 3$

$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$

$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Rightarrow a^2 + b^2 - c^2 = ab$

Now, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{ab}{2ab} = \frac{1}{2}$

$\Rightarrow \underline{|C} = 60°$

4. Observe the following statements : **[EAMCET 2008]**

I) In ΔABC $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s$

II) In $\Delta ABC, \cot\frac{A}{2} = \frac{b+c}{2} \Rightarrow B = 90°$

Which of the following is correct?

1) Both I and II are true 2) I is true, II is false

3) I is false , II is true 4) Both I and II are false

Ans: 2

Sol. I) $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}=b\frac{s(s-c)}{ab}+c\frac{s(s-b)}{ac}=\frac{s}{a}(s-c+s-b)=s$

II) If $A=45°, B=90°, C=45°$, then $a=\sqrt{2}R, b=2R, c=\sqrt{2}R$

But $\frac{b+c}{2}=\frac{2R+\sqrt{2}R}{2}=\left(\sqrt{2}+1\right)\frac{R}{\sqrt{2}}=\frac{R}{\sqrt{2}}\cot 22\frac{1}{2}°\neq\cot\frac{A}{2}$

5. In a triangle, if $r_1=2r_2=3r_3$, then $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=$ **[EAMCET 2008]**

1) $\frac{75}{60}$ 2) $\frac{155}{60}$ 3) $\frac{176}{60}$ 4) $\frac{191}{60}$

Ans: 4

Sol. $r_1=2r_2=3r_3\Rightarrow\frac{\Delta}{s-a}=\frac{2\Delta}{s-b}=\frac{3\Delta}{s-c}=\frac{\Delta}{k}(\text{say})$

$\Rightarrow s-a=k, s-b=2k, s-c=3k\Rightarrow s-a+s-b+s-c=6k$

$\Rightarrow s=6k\Rightarrow a=5k, b=4k, c=3k$

$\therefore\ \frac{a}{b}+\frac{b}{c}+\frac{c}{a}=\frac{5k}{4k}+\frac{3k}{5k}=\frac{5}{4}+\frac{4}{3}+\frac{3}{5}=\frac{75+80-36}{60}=\frac{191}{60}$

(or)

If $xr_1=yr_2=zr_3$, then

$a:b:c=y+z:z+x:x+y$

$\therefore\ a:b:c=5:4:3$

6. If two angles of ΔABC are $45°$ and $60°$, then the ratio of the smallest and the greatest sides are **[EAMCET 2007]**

1) $\left(\sqrt{3}-1\right):1$ 2) $\sqrt{3}:\sqrt{2}$ 3) $1:\sqrt{3}$ 4) $\sqrt{3}:1$

Ans: 1

Sol. Angles are 45°, 60° and 75°.

The ratio of smallest and greatest sides = sin45°; sin75°= $\sqrt{3}-1:1$

7. In ΔABC, $(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=$ **[EAMCET 2007]**

1) $2c\cot\frac{C}{2}$ 2) $2a\cot\frac{A}{2}$ 3) $2b\cot\frac{B}{2}$ 4) $\tan\frac{C}{2}$

Ans: 1

Sol. $(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=2s\left(\frac{\Delta}{s(s-a)}+\frac{\Delta}{s(s-b)}\right)$

$=\frac{2\Delta c}{(s-a)(s-b)}=\frac{2c\sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)}$

$=2c\ \cot\frac{c}{2}$

8. In ΔABC, with usual notation, observe the two statements given below **[EAMCET 2007]**

I) $rr_1r_2r_3 = \Delta^2$

II) $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Which of the following is correct

1) Both I and II are true   2) I is true, II is false
3) I is false, II is true   4) Both I and II are false

Ans: 1

Sol. i) $rr_1r_2r_3 = \frac{\Delta}{s}.\frac{\Delta}{s-a}.\frac{\Delta}{s-b}.\frac{\Delta}{s-c} = \Delta^2$

ii) $r_1r_2 + r_2r_3 + r_3r_1 = \frac{\Delta}{s-a}.\frac{\Delta}{s-b} + \frac{\Delta}{s-b}.\frac{\Delta}{s-c} + \frac{\Delta}{s-c}.\frac{\Delta}{s-a}$

$= s(s-c) + s(s-a) + s(s-b) = s^2$

9. If, in a $\Delta ABC, \tan\frac{A}{2} = \frac{5}{6}$ and $\tan\frac{C}{2} = \frac{2}{5}$ then a, b, c are such that : **[EAMCET 2006]**

1) b^2 + ac   2) 2b = a + c   3) 2ac = b(a + c)   4) a + b = c

Ans: 2

Sol. $\tan\frac{A}{2}.\tan\frac{C}{2} = \frac{5}{6}.\frac{2}{5} = \frac{1}{3}$

$\sqrt{\frac{(s-b)(s-c)}{s(s-a)}\frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{3}$

$\Rightarrow \frac{s-b}{s} = \frac{1}{3} \Rightarrow 3s - 3b = s$

2s = 3b

a + b + c = 3b

$\Rightarrow a + c = 2b$

10. The angles of a triangle are in the ratio 3 : 5 : 10. Then the ratio of the smallest side to the greatest side is **[EAMCET 2006]**

1) 1: sin 10°   2) 1 : 2 sin 10°   3) 1 : cos 10°   4) 1 : 2 cos10°

Ans: 4

Sol. Let angles as 3x, 5x, 10x

$\therefore$ $18x = 180° \Rightarrow x = 10°$

$\therefore$ angle aer 30°, 50°, 100°

a : c = sinA : sinC = sin30 : sin100

$\frac{1}{2} : \sin(90+10) = 1 : 2\cos 10°$

11. In a triangle ABC, $\frac{s-a}{\Delta} = \frac{1}{8}, \frac{s-b}{\Delta} = \frac{1}{12}, \frac{s-c}{\Delta} = \frac{1}{24}$ then b = **[EAMCET 2006]**

1) 16   2) 20   3) 24   4) 28

Ans: 1

Sol. $\frac{1}{r_1} = \frac{s-a}{\Delta} = \frac{1}{8}$; $\frac{1}{r_2} = \frac{s-b}{\Delta} = \frac{1}{12}$; $\frac{1}{r_3} = \frac{s-c}{\Delta} = \frac{1}{24}$

$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{6}{24} = \frac{1}{4}$

$(r_2 - r)(r_1 + r_3) = b^2$

$(12-4)(24+8) = b^2 \Rightarrow 16\times16, b = 16$

12. In a ΔABC, $a(\cos^2 B + \cos^2 C) + \cos A(c\cos C + b\cos B) =$ **[EAMCET 2005]**

1) a 2) b 3) c 4) a + b +c

Ans: 1

Sol. $= a\left[\left(\frac{a^2+c^2-b^2}{2ac}\right)^2 + \left(\frac{a^2+b^2-c^2}{2ab}\right)^2\right]$

$+\left(\frac{b^2+c^2-a^2}{2bc}\right)\left[c\left(\frac{b^2+a^2-c^2}{2ab}\right) + b\left(\frac{a^2+c^2-b^2}{2ac}\right)\right]$

$\frac{a^2+c^2-b^2}{2a} + \frac{a^2+b^2-c^2}{2a} = \frac{2a^2}{2a} = a$

13. In a ΔABC, $\sum(b+c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right) =$ **[EAMCET 2005]**

1) a 2) b 3) c 4) 0

Ans: 4

Sol. From Napier's formula $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\Sigma(b+c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right) = \Sigma(b+c)\tan\frac{A}{2}.\frac{b-c}{b+c}\cot\frac{A}{2}$

$\Sigma(b-c) = 0$

14. Two sides of a triangle are given by the roots of the equation $x^2 - 5x + 6 = 0$ and the angle between the sides is $\pi/3$. Then the perimeter of the triangle is **[EAMCET 2005]**

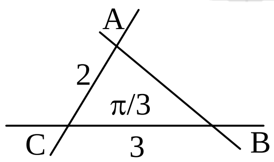
1) $5+\sqrt{2}$ 2) $5+\sqrt{3}$ 3) $5+\sqrt{5}$ 4) $5+\sqrt{7}$

Ans: 4

Sol. $c^2 = a^2 + b^2 - 2ab\cos C$

$= 9 + 4 - 12\times\frac{1}{2} = 7$

$= c = \sqrt{7}$



$2S = a + b + c = 5 + \sqrt{7}$

15. If , in a $\Delta ABC, r_3 = r_1 + r_2 + r$, then $\underline{|A} + \underline{|B} =$ **[EAMCET 2004]**

1) 120° 2) 100° 3) 90° 4) 80°

Ans: 3

Sol. $r_3 - r = 4R\sin^2\frac{c}{2}$

$r_1 + r_2 = 4R\cos^2\frac{c}{2}$

$r_1 + r_2 + r - r_3 = 0$

$\Rightarrow 4R\left(\cos^2\frac{c}{2}-\sin^2\frac{c}{2}\right)=0$

$\Rightarrow 4R\cos c=0\Rightarrow c=90°$

$\therefore\ \underline{|A}+\underline{|B}=90°$

16. In a $\Delta ABC,(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin\frac{C}{2}=$ **[EAMCET 2004]**

1) a^2 2) c^2 3) b^2 4) a^2+b^2

Ans: 2

Sol. $a^2+b^2-2ab\left(\cos^2\frac{c}{2}-\sin^2\frac{c}{2}\right)=c^2$

17. In a ΔABC, the correct formulae among the following are **[EAMCET 2004]**

I) $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

II) $r_1=(s-a)\tan\frac{A}{2}$

III) $r_3=\frac{\Delta}{s-c}$

1) only I, II 2) only II, III 3) only I, III 4) I, II, III

Ans: 3

Sol. I is true, II is false, III is true

18. If in a ΔABC, $r_1<r_2<r_3$ then **[EAMCET 2003]**

1) a < b < c 2) a > b > c 3) b < a < c 4) a < c < b

Ans: 1

Sol. $r_1<r_2<r_3$

$\Rightarrow\frac{\Delta}{S-a}<\frac{\Delta}{S-b}<\frac{\Delta}{S-c}$

$\Rightarrow S-a>S-b>S-c$

$\Rightarrow -a>-b>-c$

$\therefore\ a<b<c$

19. If in a ΔABC, if $3a=b+c$, then $\cot\frac{B}{2}.\cot\frac{C}{2}=$ **[EAMCET 2003]**

1) 1 2) 2 3) 3 4) 4

Ans: 2

Sol. $\cot\frac{B}{2}\cot\frac{C}{2}=\frac{S(S-b)}{\Delta}\times\frac{S(S-c)}{\Delta}$

$=\frac{S}{S-a}=\frac{2S}{2(S-a)}=\frac{a+b+c}{b+c-a}=2$

20. In a ΔABC, if b = 20, c = 21 and sinA = 3/5, then a = ……… **[EAMCET 2003]**

1) 12 2) 13 3) 14 4) 15

Ans: 2

Sol. $\sin A=\frac{3}{5}\Rightarrow\cos A=\frac{4}{5}$

$a^2=b^2+c^2-2bc\cos A$

a = 13

21. If ΔABC is right angled at A, then $r_2 + r_3 =$ **[EAMCET 2002]**

1) $r_1 - r$ 2) $r_1 + r$ 3) $r - r_1$ 4) R

Ans: 1

Sol. $\underline{|A} = 90°$ and $r_2 + r_3$

$= 4R\cos^2\frac{A}{2} = 2R$

$r_2 - r = 4R\sin^2\frac{A}{2} = 2R$

22. In a $\Delta ABC, \frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b} =$ **[EAMCET 2001]**

1) $\frac{1}{a}$ 2) $\frac{1}{b}$ 3) $\frac{1}{c}$ 4) $\frac{c+a}{b}$

Ans: 2

Sol. $\frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b} =$

$= \frac{b\cos C + b\cos A + c\cos B + a\cos B}{b(c+a)}$

$\frac{(c+a)}{b(c+a)} = \frac{1}{b}$

23. In a ΔABC, $a^2\sin 2C + c^2\sin 2A$ **[EAMCET 2001]**

1) Δ 2) 2Δ 3) 3Δ 4) 4Δ

Ans: 4

Sol. $a^2\sin 2C + c^2\sin 2A$

$= 4R^2\sin^2 A.2\sin C\cos C + 4R^2\sin^2 C.2\sin A\cos A$

$= 8R^2\sin A\sin C(\sin A\cos C + \cos A\sin C)$

$= 8R^2\sin A\sin B\sin C = 4\Delta$

24. If in a ΔABC, a, b, c are in arithmetic progression, then the (A/2) tan (C/2) = **[EAMCET 2000]**

1) 1/4 2) 1/3 3) 3 4) 4

Ans: 2

Sol. $\tan\frac{A}{2}\tan\frac{C}{2} = \frac{\Delta}{s(s-a)}.\frac{\Delta}{s(s-c)}$

$= \frac{s-b}{s} = \frac{2(s-b)}{2s}$

$= \frac{a+c-b}{a+b+c} = \frac{2b-b}{2b+b} = \frac{1}{3}$

25. If a ΔABC, $\cos A + \cos B + \cos B + \cos C =$ **[EAMCET 2000]**

1) $1+\frac{r}{R}$ 2) $1-\frac{r}{R}$ 3) $1-\frac{R}{r}$ 4) $1+\frac{R}{r}$

Ans: 1

Sol. $\cos A + \cos B + \cos C = 1 + 4R\ \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$$= 1 + \frac{4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{R} = 1 + \frac{r}{R}$$

26. In a ΔABC, $r + r_3 + r_1 - r_2 =$ **[EAMCET 2000]**

1) 4R cosA 2) 4R cosB 3) 4R cosc 4) 4R

Ans: 2

Sol. $r_1 + r_3 = 4R\cos^2\frac{B}{2}$

$r_2 - r_1 = 4R\sin^2\frac{B}{2}$

$r + r_1 + r_3 - r_2 = 4R\left(\cos^2\frac{B}{2} - \sin^2\frac{B}{2}\right) = 4R\cos B$