PERIODICITY AND EXTREME VALUES

PREVIOUS EAMCET BITS

1. The period of $\sin^4 x + \cos^4 x$ is **[EAMCET 2009]**

   1) $\frac{\pi^4}{2}$ 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

   Ans: 4

Sol. $f\left(\frac{\pi}{2}+x\right) = \cos^4 x + \sin^4 x = f(x)$

$\therefore$ Period = $\pi/2$

2. For all values of θ, the values of $3 - \cos\theta + \cos\left(\theta + \frac{\pi}{3}\right)$ lie in the interval **[EAMCET 2006]**

   1) $[-2,3]$ 2) $[-2,1]$ 3) $[2,4]$ 4) $[1,5]$

   Ans: 3

Sol. $\cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta\cos\frac{\pi}{3} - \sin\theta.\sin\frac{\pi}{3}$

$= \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$3 - \cos\theta + \cos\left(\theta+\frac{p}{3}\right) = 3 - \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

Min. value = $C - \sqrt{a^2+b^2} = 3 - \sqrt{\frac{1}{4}+\frac{3}{4}} = 2$

Max. value = $C + \sqrt{a^2+b^2} = 3+1 = 4$

3. The extreme value of $4\cos\left(x^2\right)\cos\left(\frac{\pi}{3}+x^2\right)\cos\left(\frac{\pi}{3}-x^2\right)$ over $\mathbb{R}$ are **[EAMCET 2005]**

   1) – 1, 1 2) –2, 2 3) –3, 3 4) –4, 4

   Ans: 1

Sol. $\cos A\cos(60-A)\cos(60+A) = \frac{1}{4}\cos 3A$

$\therefore\ 4\cos x^2\cos\left(\frac{\pi}{3}+x^2\right)\cos\left(\frac{\pi}{3}-x^2\right) = \cos 3x^2 = [-1\ 1]$

4. If $n \in N$, and the period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is 4π, then n = **[EAMCET 2004]**

   1) 4 2) 3 3) 2 4) 1

   Ans: 3

Sol. Period of $\cos nx = \frac{2\pi}{n}$

Period of $\sin\left(\frac{x}{n}\right) = 2n\pi$

$2n\pi = 4\pi \Rightarrow n = 2$

5. For $x \in \mathbb{R}, 3\cos(4x-5)+4$ lies in the interval **[EAMCET 2004]**

1) $[1,7]$ 2) $[4,7]$ 3) $[0,7]$ 4) $[2,7]$

Ans: 1

Sol. Maximum and Minimum values of $\cos\theta$ are 1 an – 1

$\therefore\ [-3+4, 3+4] = [1,7]$

6. The period of the function $f(\theta) = \sin\frac{\theta}{3} + \cos\frac{\theta}{2}$ is **[EAMCET 2003]**

1) 3π 2) 6π 3) 9π 4) 12π

Ans: 4

Sol. The period of $\sin\left(\frac{\theta}{3}\right)$ is $\frac{2\pi}{1/3} = 6\pi$

The period of $\cos\frac{\theta}{2}$ is $\frac{2\pi}{1/2} = 4\pi$

The period of $\sin\left(\frac{\theta}{3}\right) + \cos\left(\frac{\theta}{2}\right)$ is L.C.M of $6\pi, 4\pi = 12\pi$

7. If $f(x) = \sin^2\left(\frac{\pi}{8}+\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{\pi}{2}\right)$, then the period of f is **[EAMCET 2002]**

1) π 2) $\pi/2$ 3) $\pi/3$ 4) 2π

Ans: 4

Sol. $\text{Sin}^2A - \sin^2 B = \sin(A+B)\text{Sin}(A-B)$

$\therefore\ \sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right) = \sin\frac{\pi}{4}\sin x$

$\therefore$ period = 2π

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