

PARTIAL DIFFERENTIATION

PREVIOUS EAMCET BITS

1. $z = \tan(y + ax) + \sqrt{y - ax} \Rightarrow z_{xx} - a^2 z_{yy} =$ **[EAMCET 2009]**
 1) 0 2) 2 3) $z_x + z_y$ 4) $z_x z_y$

Ans: 1

Sol. $Z = f(y + ax) + g(y - ax)$
 $\Rightarrow Z_{xx} - a^2 Z_{yy} = 0$

2. If $z = \sec^{-1}\left(\frac{x^4 + y^4 - 8x^2y^2}{x^2 + y^2}\right)$ then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} =$ **[EAMCET 2008]**
 1) $\cot z$ 2) $2 \cot z$ 3) $2 \tan z$ 4) $2 \sec z$

Ans: 2

Sol. $z = z = \sec^{-1}\left(\frac{x^4 + y^4 - 8x^2y^2}{x^2 + y^2}\right) \Rightarrow \sec z$ is a homogeneous function of degree 2
 $\Rightarrow x \frac{\partial}{\partial x} [\sec z] + y \frac{\partial}{\partial y} [\sec z] = 2 \sec z \Rightarrow x \sec z \tan z \frac{\partial z}{\partial x} + y \sec z \tan z \frac{\partial z}{\partial y} = 2 \sec z$
 $\Rightarrow x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2 \cot z$

3. If $z = \log(\tan x + \tan y)$, then $(\sin 2x) \frac{\partial z}{\partial x} + (\sin 2y) \frac{\partial z}{\partial y} =$ **[EAMCET 2007]**
 1) 1 2) -2 3) 3 4) 4

Ans: 2

Sol. $\frac{\partial z}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y}$
 $\frac{\partial z}{\partial y} = \frac{\sec^2 y}{\tan x + \tan y}$
 $\sin 2x \frac{\partial z}{\partial x} + \sin 2y \frac{\partial z}{\partial y} = 2$

4. $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right) \Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ **[EAMCET 2006]**
 1) $\sin u$ 2) $\tan u$ 3) $\cos u$ 4) $\cot u$

Ans: 2

Sol. $\sin u = \frac{x^2 + y^2}{x + y} = f(x, y)$
 $f(x, y)$ is a Homogeneous equation of '1' degree
 \therefore by Euler's theorem

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n.f. = f(x, y)$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n. \frac{f(u)}{f'(u)}$$

$$\Rightarrow \frac{\sin u}{\cos u} = \tan u$$

5 $z = \cos^{-1}(x+y) + \sec^{-1}(y+2x) \Rightarrow \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial y^2} =$ [EAMCET 2005]

- 1) $\frac{3\partial^2 z}{\partial x \partial y}$ 2) $\frac{-\partial^2 z}{\partial x \partial y}$ 3) $\frac{-3\partial^2 z}{\partial x \partial y}$ 4) $\frac{\partial^2 z}{\partial x \partial y}$

Ans: 1

Sol. differentiating partially

6. $f(x, y) = 2(x-y)^2 - x^4 - y^4 \Rightarrow (f_{xx}f_{yy} - f_{xy}^2)_{(0,0)}$ [EAMCET 2004]

- 1) 32 2) 16 3) 0 4) -1

Ans: 3

Sol. $f(x, y) = 2(x-y)^2 - x^4 - y^4$

$$(f_{xx})_{(0,0)} = 4 - 12x^2 = 4$$

$$(f_{yy})_{(0,0)} = -4 - 12y^2$$

$$(f_{xy})_{(0,0)} = -4$$

$$\therefore (f_{xx}f_{yy} - f_{xy}^2)_{(0,0)} = 4 \times 4 - 4^2 = 0$$

7. If $u(x, y) = y \log x + x \log y$, then $u_x u_y - u_x \log x - u_y \log y + \log x \log y = \dots$ [EAMCET 2003]

- 1) 0 2) -1 3) 1 4) 2

Ans: 3

Sol. $u(x, y) = y \log x + x \log y$

$$u_x = \frac{y}{x} + \log y \text{ and } u_y = \log x + \frac{x}{y}$$

$$u_x u_y - u_x \log x - u_y \log y + \log x \log y$$

$$= (u_x - \log y)(u_y - \log x)$$

$$= \frac{y}{x} \cdot \frac{x}{y} = 1$$

8. If $z = \frac{y}{x} \left[\sin \frac{x}{y} + \cos \left(1 + \frac{y}{x} \right) \right]$, then $x \frac{\partial z}{\partial x} =$ [EAMCET 2002]

- 1) $y \frac{\partial z}{\partial y}$ 2) $-y \frac{\partial z}{\partial y}$ 3) $2y \frac{\partial z}{\partial y}$ 4) $2y \frac{\partial z}{\partial x}$

Ans: 2

Sol. $z = \frac{y}{x} \left[\sin \frac{x}{y} + \cos \left(1 + \frac{y}{x} \right) \right]$ then

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0 \Rightarrow x \frac{\partial z}{\partial x} = -y \frac{\partial z}{\partial y}$$

9. If $z = \sec(y - ax) + \tan(y + ax)$, then $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} =$

[EAMCET 2002]

- 1) z 2) $2z$ 3) 0 4) $-z$

Ans: 3

Sol. $z = \sec(y - ax) + \tan(y + ax)$

$$z = f(ax + y) + \phi(y - ax)$$

$$\frac{\partial^2 z}{\partial x^2} + a^2 \frac{\partial^2 z}{\partial y^2}$$

$$\therefore \frac{\partial^2 z}{\partial x^2} + a^2 \frac{\partial^2 z}{\partial y^2} \Rightarrow \frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = 0$$

10. If $u = e^{-x^2 - y^2}$, then

[EAMCET 2001]

- 1) $xu_x = yu_y$ 2) $yu_x = xu_y$ 3) $yu_x + xu_y = 0$ 4) $x^2u_y + y^2u_x = 0$

Ans: 2

Sol. $u = e^{-x^2 - y^2}$

$$ux = -2xe^{-x^2 - y^2}; uy = -2ye^{-x^2 - y^2}$$

$$\therefore yu_x = xu_y$$

11. If $u = xy^2 \tan^{-1} \left(\frac{y}{x} \right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

[EAMCET 2001]

- 1) $2u$ 2) u 3) $3u$ 4) $\frac{1}{3}u$

Ans: 3

Sol. Given $u = xy^2 \cdot \tan^{-1} \left(\frac{y}{x} \right)$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu = 3u (\because n = 3)$$

12. If $u = \log_e(x^2 + y^2) + \tan^{-1} \left(\frac{y}{x} \right)$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} =$

[EAMCET 2000]

- 1) 0 2) $2u$ 3) $1/u$ 4) u

Ans: 1

Sol. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

13. If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

[EAMCET 2000]

1) $\frac{1}{2} \cot u$

2) $2 \cot u$

3) $-\frac{1}{2} \cot u$

4) $3 \cot u$

Ans: 3

Sol. $n = \frac{1}{2} \Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -n \cot u$
 $= -\frac{1}{2} \cot u$

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