PAIR OF STRAIGHT LINES

PREVIOUS EAMCET BITS

1. The value of λ with $|\lambda| < 16$ such that $2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$ represents a pair of straight lines, is **[EAMCET 2009]**

   1) -10 2) -9 3) 10 4)9

   Ans: 2

Sol. $\Delta = 0 \Rightarrow \lambda = -9$

2. The area (in square units) of the triangle formed by $x + y + 1 = 0$ and the pair of straight lines $x^2 - 3xy + 2y^2 = 0$ is **[EAMCET 2009]**

   1) 7/12 2) 5/12 3) 1/12 4) 1/6

   Ans: 3

Sol. $\text{Area} = \dfrac{n^2\sqrt{h^2 - ab}}{\left|am^2 - 2h\ell m + b\ell^2\right|} = \dfrac{1}{12}$

3. The pairs of straight lines $x^2 - 3xy + 2y^2 = 0$ and $x^2 - 3xy + 2y^2 + x - 2 = 0$ form a **[EAMCET 2009]**

   1) square but not rhombus 2) rhombus

   3) parallelogram 4) rectangle but not a square

   Ans: 3

Sol. In given two equations homogenious parts are same, hence is a parallelogram.

4. The value of λ such that $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, is **[EAMCET 2008]**

   1) 1 2) -1 3) 2 4) -2

   Ans: 3

Sol. $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0 \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Rightarrow a = \lambda, b = 12, c = -3, f = -8, g = 5/2, h = -5$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow \lambda(12)(-3) + 2(-8)(5/2)(-5) - \lambda(-8)^2 - 12(5/2)^2 - (-3)(-5)^2 = 0$

$\Rightarrow -36\lambda + 200 - 64\lambda - 75 + 75 = 0 \Rightarrow 100\lambda = 200 \Rightarrow \lambda = 2$

5. A pair of perpendicular straight lines passes through the origin and also through the point of intersection of the curve $x^2 + y^2 = 4$ with $x + y = a$. The set containing the value of 'a' is **[EAMCET 2008]**

   1) $\{-2, 2\}$ 2) $\{-3, 3\}$ 3) $\{-4, 4\}$ 4) $\{-5, 5\}$

   Ans: 1

Sol. Clearly the point of intersection of $x^2 + y^2 = 4, x + y = a$ is the origin

$x^2+y^2=4, x+y=a \Rightarrow x^2+(a-x)^2=4, (x=0) \Rightarrow a^2=4 \Rightarrow a=\pm 2$

6. The angle between the pair of straight lines formed by joining the points of intersection of $x^2+y^2=4$ and $y=3x+c$ to the origin is a right angle. Then $c^2=$ **[EAMCET 2007]**

1) 20 2) 13 3) 1/5 4) 5

Ans: 1

Sol. $2c^2=a^2(1+m^2)$

$\Rightarrow 2c^2=40 \Rightarrow c^2=20$

7. If the lines $x^2+2xy-35y^2-4x+44y-12=0$ and $5x+\lambda y-8=0$ are cocurrent, then the value of λ is **[EAMCET 2007]**

1) 0 2) 1 3) – 1 4) 2

Ans: 4

Sol. Point of intersection of given pair of lines is $\left(\frac{4}{3},\frac{2}{3}\right)$

It lies on $5x+\lambda y-8=0 \Rightarrow \lambda=2$

8. The lines represented by the equation $x^2-y^2-x+3y-2=0$ are **[EAMCET 2006]**

1) $x+y-1=0, x-y+2=0$ 2) $x-y-2=0, x+y+1=0$

3) $x+y+2=0, x-y-1=0$ 4) $x-y+1=0, x+y-2=0$

Ans: 4

Sol. By verification

$(x-y+1)(x+y-2)=0$

$\Rightarrow x^2+y^2-x+3y-2=0$

9. The centroid of the triangle formed by the pair of straight lines $12x^2 - 20xy + 7y^2 = 0$ and the line $2x - 3y + 4 = 0$ is **[EAMCET 2006]**

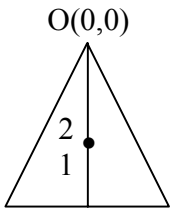
1) $\left(-\frac{7}{3},\frac{7}{3}\right)$ 2) $\left(-\frac{8}{3},\frac{8}{3}\right)$ 3) $\left(\frac{8}{3},\frac{8}{3}\right)$ 4) $\left(\frac{4}{3},\frac{4}{3}\right)$

Ans: 3

Sol. Centroid divides median in the ratio of 2 : 1

$\therefore$ of (x_1,y_1) is centroid then $\left(\frac{3x_1}{2},\frac{3y_1}{2}\right)$ must lie on

$2x - 3y + 4 = 0$ by verification $\left(\frac{8}{3},\frac{8}{3}\right)$ is centroid

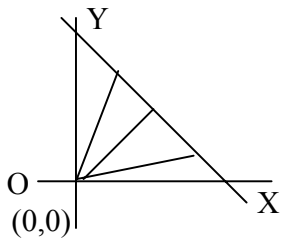


10. The area of the triangle formed by the pair of straight lines $(ax + by)^2 - 3(bx - ay)^2 = 0$ and $ax + by + c = 10$ is **[EAMCET 2005]**

1) $\frac{c^2}{a^2+b^2}$ 2) $\frac{c^2}{2\left(a^2+b^2\right)}$ 3) $\frac{c^2}{\sqrt{2}\left(a^2+b^2\right)}$ 4) $\frac{c^2}{\sqrt{3}\left(a^2+b^2\right)}$

Ans: 4

Sol.



$\perp^{lar}$ distance from 0(0, 0) to ax + by + c = 0 is

$h = \frac{c}{\sqrt{a^2+b^2}}$

Area $\frac{h^2}{\sqrt{3}} = \frac{c}{\sqrt{3}\left(a^2+b^2\right)}$

11. The product of the perpendicular distances from the origin on the pair of straight lines $12x^2+25xy+12y^2+10x+11y+2=0$ is **[EAMCET 2005]**

1) $\frac{1}{25}$ 2) $\frac{2}{25}$ 3) $\frac{3}{25}$ 4) $\frac{4}{25}$

Ans: 2

Sol. Product of perpendiculars $= \frac{|c|}{\sqrt{(a-b)^2+(2h)^2}} = \frac{2}{25}$

12. The angle between the lines represented by $y^2\sin^2\theta - xy\sin^2\theta + x^2\left(\cos^2\theta - 1\right) = 0$ is **[EAMCET 2004]**

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

Ans: 4

Sol. $a+b=0 \Rightarrow \theta = 90°$

13. Area of the triangle formed by the lines $3x^2-4xy+y^2=0, 2x-y=6$ is **[EAMCET 2004]**

1) 16 2) 25 3) 36 4) 49

Ans: 3

Sol. Area of the triangle $= \left|\frac{n^2\sqrt{h^2-ab}}{am^2-2h\ell m+b\ell^2}\right| = 36$ sq.units

14. If the pair of straight lines given by $Ax^2+2Hxy+By^2=0\left(H^2>AB\right)$ forms an equilateral triangle with line ax + by + c = 0, then (A + 3B) (3A + B) = ….. **[EAMCET 2003]**

1) H^2 2) $-H^2$ 3) $2H^2$ 4) $4H^2$

Ans: 4

Sol. Equation of one side ax + by + c =0 and other two sides passing through origin then the equation of other two sides is $(ax+by)^2-3(bx-ay)^2=0$

$A = a^2 - 3b^2$; $B = b^2 - 3a^2$; $H = 8ab$

$\therefore\ (A+3B)(3A+B)=4H^2$

15. The area (in square units) of the quadrilateral formed by the two pairs of lines $\ell^2x^2-m^2y^2-n(\ell x+my)=0$ and $\ell^2x^2-m^2y^2+n(\ell zx-my)=0$ **[EAMCET 2003]**

1) $\frac{n^2}{2|\ell.m|}$ 2) $\frac{n^2}{|\ell.m|}$ 3) $\frac{n^2}{|\ell.m|}$ 4) $\frac{n^2}{4|\ell.m|}$

Ans: 1

Sol. $\ell^2x^2-m^2y^2-n(\ell x+my)=0$

$\Rightarrow(\ell x+my)(\ell x-my-n)=0$

$\ell^2x^2-m^2y^2+n(\ell x-my)=0$

$\Rightarrow(\ell x-my)(\ell x+my+n)=0$

The given lines from a rhombus

$\therefore$ Area of rhombus $=\frac{c^2}{2|ab|}=\frac{n^2}{2|\ell m|}$

16. If the coordinate axes are the bisectors of the angle between the pair of lines $ax^2+2hxy+b^2=0$, where $h^2 > ab$ and $a \neq b$, then **[EAMCET 2002]**

1) $a+b=0$ 2) $h=0$ 3) $h\neq 0, a+b=0$ 4) $a+b\neq 0$

Ans: 2

Sol. Equation of pair of angular bisectors is $h(x^2-y^2)-(a-b)xy=0$

Equation of coordinate axes is xy = 0

$\therefore$ h = 0

17. If the angle 2θ is acute then the acute angle between the pair of straight lines $x^2(\cos\theta - \sin\theta) + 2xy\cos\theta + y^2(\cos\theta + \sin\theta) = 0$ **[EAMCET 2002]**

1) 2θ 2) θ/2 3) θ/3 4) θ

Ans: 4

Sol. $\cos\alpha=\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$

(α is angle between the pair of lines and $a = \cos\theta + \sin\theta$, $b = \cos\theta - \sin\theta$ $2h = 2\cos\theta$)

$\cos\alpha = \cos\theta$

$\therefore$ Angle 'θ'

18. If the pair of straight lines $xy - x - y + 1 = 0$ and the line $ax + 2y - 3 = 0$ are concurrent, then a = **[EAMCET 2002]**

1) -2 2) 3 3) 1 4) 0

Ans: 3

Sol. $xy - x - y + 1 = 0$

$\Rightarrow (x-1)(y-1) = 0 \ \therefore \ (x, y) = (1,1)$

$ax + 2y - 3 = 0$ line passes through (1, 1)

$\therefore a = 1$

19. The orthocentre of the triangle formed by the lines $x + 3y = 10$ and $6x^2 + xy - y^2 = 0$ is **[EAMCET 2001]**

1) (1, 3) 2) (3, 1) 3) (–1, 3) 4) (1, –3)

Ans: 1

Sol. The given lines are $x + 3y = 10$, $6x^2 + xy - y^2 = 0$

$\ell = 1, m = 3, n = 10, a = 6, h = \frac{1}{2}; b = -1$

$\therefore$ Orthocentre = $((k\ell; km)$ where $k = \frac{n(a+b)}{am^2 - 2h\ell m + b\ell^2}$

20. If one of the lines of the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the co-ordinate axes then : **[EAMCET 2001]**

1) $a^2 + b^2 = h^2$ 2) $(a+b)^2 = h^2$ 3) $a^2 + b^2 = 4h^2$ 4) $(a+b)^2 = 4h^2$

Ans: 4

Sol. The angle bisectors of the axes are $x \pm y = 0$

$ax^2 \pm 2hx^2 + bx^2 = 0$

$\Rightarrow (a+b)^2 = 4h^2$

21. If the slope of one line is twice the slope of the other in the pair of straight lines $ax^2 + 2hxy + by^2 = 0$, then $8h^2 =$ **[EAMCET 2001]**

1) $-9ab$ 2) $9ab$ 3) $7ab$ 4) $-7ab$

Ans: 2

Sol. If slope of one line is 'k' times to other then

$4kh^2 = (k+1)^2 ab \Rightarrow 8h^2 = 9ab$

22. The equation of the pair of lines through the point (a, b) parallel to the coordinate axes is **[EAMCET 2000]**

1) $(x-b)(y-a) = 0$ 2) $(x-b)(y+b) = 0$

3) $(x-a)(y-b) = 0$ 4) $(x+a)(y-b) = 0$

Ans: 3

Sol. The equation of the coordinate axes is $xy = 0$

$\therefore$ The equation of the pair of lines passing through (a, b) and parallel to xy = 0 is (x – a) (y – b) = 0

☆☆☆