

## MAXIMA AND MINIMA

### PREVIOUS EAMCET BITS

1. The function  $f(x) = x^3 + ax^2 + bx + c, a^2 \leq 3b$  has [EAMCET 2009]

- 1) one maximum value 2) one minimum value  
 3) no extreme value 4) one maximum and one minimum value

Ans: 3

Sol.  $f(x) = x^3 + ax^2 + bx + c$

$$f'(x) = 3x^2 + 2ax + b$$

$$\therefore \Delta = 4a^2 - 12b = 4(a^2 - 3b) < 0$$

$\therefore f'(x)$  never zero.

2. The maximum value of  $\frac{\log x}{x}, 0 < x < \infty$  is [EAMCET 2009]

- 1)  $\infty$  2) e 3) 1 4)  $e^{-1}$

Ans: 4

Sol.  $f(x) = \frac{\log x}{x}$

$$\therefore f'(x) = \frac{x \left( \frac{1}{x} \right) - \log x (1)}{x^2} = 0 \Rightarrow f'(x) = \frac{1 - \log x}{x} = 0$$

$$\Rightarrow x = e$$

$$\therefore \text{Max. } f = \frac{1}{e}$$

3. If m and M respectively denote the minimum and maximum of  $f(x) = (x-1)^2 + 3$  for  $x \in [-3, 1]$  then the order pair (m, M) [EAMCET 2008]

- 1) (-3, 19) 2) (3, 19) 3) (-19, 3) 4) (-19, -3)

Ans: 2

Sol.  $f(x) = (x-1)^2 + 3 \Rightarrow f'(x) = 2(x-1) \Rightarrow f''(x) = 2 > 0$

$$f'(x) = 0 \Rightarrow x = 1. \text{ Minimum value} = f(1) = 3$$

$$-3 \leq x \leq 1 \Rightarrow -4 \leq x-1 \leq 0 \Rightarrow (x-1)^2 + 3 \leq 19$$

$$\text{Minimum value} = 3 \therefore (m, M) = (3, 19)$$

4. The condition for  $f(x) = x^3 + px^2 + qx + r (x \in \mathbb{R})$  to have no extreme value, is [EAMCET 2007]

- 1)  $p^2 < 3q$  2)  $2p^2 < q$  3)  $p^2 < 1/4q$  4)  $p^2 > 3q$

Ans: 1

Sol.  $f(x)$  has no extreme value then  $f'(x) = 0$  have no real roots.

$$f'(x) = 3x^2 + 2px + q$$

$$4p^2 - 12q < 0 \Rightarrow p^2 < 3q$$

5. Observe the statements given below:

[EAMCET 2007]

Assertion (A) :  $f'(x) = xe^{-x}$  has the maximum at  $x = 1$

Reason (R) :  $f'(1) = 0$  and  $f''(1) < 0$

Which of the following is correct ?

- 1) Both A and R are true and R is the correct reason for A
- 2) Both A and R are true, but R is not the correct reason for A
- 3) A is true, R is false
- 4) A is false, R is true

Ans: 1

Sol.  $f(x) = xe^{-x} \Rightarrow f'(x) = (1-x)e^{-x}$

$$f'(x) = 0 \Rightarrow x = 1$$

$$f''(x) = (2-x)e^{-x} \Rightarrow f''(1) = -\frac{1}{e} < 0$$

$\therefore f(x)$  has maximum at  $x = 1$

6. In the interval  $(-3, 3)$  the function  $f(x) = \frac{x}{3} + \frac{3}{x}$ ,  $x \neq 0$  is

- 1) increasing
- 2) decreasing
- 3) neither increasing nor decreasing
- 4) partly increasing and partly decreasing

Ans:

Sol.  $f(x) = \frac{x}{3} + \frac{3}{x}$ ,  $x \neq 0$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2} = 0$$

$$f'(x) = \frac{x^2 - 9}{3x^2}$$

$$x^2 - 9 < 0, \forall x \in (-3, 3)$$

$$\text{i.e., } f'(x) < 0 \forall x \in (-3, 3)$$

$\therefore$  decreasing

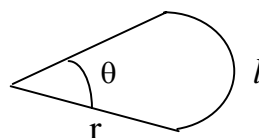
7. The perimeter of a sector is a constant. If its area is to be maximum, the sectorial angle is

[EAMCET 2006]

- 1)  $\frac{\pi^c}{6}$
- 2)  $\frac{\pi^c}{4}$
- 3)  $4^c$
- 4)  $2^c$

Ans: 4

Sol.  $\ell + 2r = k$  (say)



but  $\ell = r\theta$

$r\theta + 2r = k$

$$r = \frac{k}{\theta + 2}$$

Area  $A = \frac{1}{2}r^2\theta = \frac{k^2\theta}{2(\theta + 2)^2}$

$$A' = \frac{k^2}{2} \left\{ \frac{2(\theta + 2)\theta - (\theta + 2)^2}{(\theta + 2)^4} \right\} = 0$$

$\Rightarrow \theta = 2^\circ$

8. Observe the following statements :

[EAMCET 2005]

A :  $f'(x) = 2x^3 - 9x^2 + 12x - 3$  is increasing in  $[-\infty, 1] \cup [2, \infty]$

R :  $f'(x) < 0$  for  $x \in (1, 2)$

Then which of the following is true?

- 1) Both A and R are true and R is the correct reason for A
- 2) Both A and R are true but R is not the correct reason for A
- 3) A is true, R is false
- 4) A is false, R is true

Ans: 4

Sol. A :  $f'(x) = 6x^2 - 18x + 12 > 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

$\therefore$  A is false

B :  $f'(x) < 0, \forall x \in (1, 2)$  True

9. The minimum value of  $2x^2 + x - 1$  is

[EAMCET 2003]

- 1)  $\frac{1}{4}$                       2)  $\frac{3}{2}$                       3)  $-\frac{9}{8}$                       4)  $\frac{9}{4}$

Ans: 3

Sol. Min. value =  $\frac{4ac - b^2}{4a} = \frac{-9}{8}$

10. If  $\log(1+x) - \frac{2x}{2+x}$  is increasing then .....

[EAMCET 2002]

- 1)  $0 < x < \infty$               2)  $-\infty < x < 0$               3)  $-\infty < x < \infty$               4)  $1 < x < 2$

Ans: 1

Sol.  $f(x) = \log(1+x) - \frac{2x}{2+x}$

$$f'(x) = \frac{1}{1+x} - \frac{(2+x)^2 - 2x(1)}{(2+x)^2} > 0$$

$$\Rightarrow \frac{x^2}{(1+x)(2+x)^2} > 0 \quad \therefore x > 0$$

$$\Rightarrow 0 < x < \infty$$

11. The minimum value of  $(x - \alpha)(x - \beta)$  is [EAMCET 2001]

- 1) 0                      2)  $\alpha\beta$                       3)  $\frac{1}{4}(\alpha - \beta)^2$                       4)  $\frac{-1}{4}(\alpha - \beta)^2$

Ans: 4

Sol.  $x^2 - (\alpha + \beta)x + \alpha\beta$

The min. value of  $ax^2 + bx + c$  is  $\frac{4ac - b^2}{4a}$

$$\therefore \frac{4\alpha\beta - (\alpha + \beta)^2}{4} = \frac{-(\alpha - \beta)^2}{4}$$

12. The maximum value of  $xy$  subject to  $x + y = 7$  is [EAMCET 2001]

- 1) 12                      2) 10                      3)  $49/4$                       4)  $55/4$

Ans: 3

Sol. Given  $x + y = 7$

$$xy = x(7 - x)$$

$$\text{Let } f(x) = x(7 - x) \Rightarrow f'(x) = 7 - 2x = 0$$

$$\Rightarrow x = \frac{7}{2}; y = \frac{7}{2}$$

13. The minimum value of  $x^x$  is [EAMCET 2000]

- 1)  $e$                       2)  $e^e$                       3)  $e^{-1/e}$                       4)  $e^{1/e}$

Ans:

Sol.  $y = x^x \Rightarrow \log y = x \log x \Rightarrow \frac{dy}{dx} = x^x (1 + \log x)$

$$(1 + \log x) = 0 \Rightarrow x = e^{-1}$$

$$\therefore \text{Min. value} = e^{-1/e}$$

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