

MATHEMATICAL INDUCTION

PREVIOUS EAMCET BITS

1. Using mathematical induction, the numbers a_n 's are defined by $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$ ($n \geq 0$), then $a_n =$ [EAMCET 2009]

1) $n^3 + n^2 + 1$ 2) $n^3 - n^2 + 1$ 3) $n^3 - n^2$ 4) $n^3 + n^2$

Ans: 2

Sol. $a_0 = 1, a_1 = 1, a_2 = 3 + 1 + a_1 = 5$ and so on. Verify (2) is correct

2. For any integer $n \geq 1$, then sum $\sum_{k=1}^n k(k+2)$ is equal to [EAMCET 2008]

1) $\frac{n(n+1)(n+2)}{6}$ 2) $\frac{n(n+1)(2n+1)}{6}$ 3) $\frac{n(n+1)(2n+7)}{6}$ 4) $\frac{n(n+1)(2n+9)}{6}$

Ans: 3

Sol. $\sum_{k=1}^n k(k+2) = \sum_{k=1}^n (k^2 + 2k) = \sum_{k=1}^n k^2 + 2\sum_{k=1}^n k = \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$
 $\frac{n(n+1)}{6} [2n+1+6] = \frac{n(n+1)(2n+7)}{6}$

3. If $S_n = 1^3 + 2^3 + \dots + n^3$ and $T_n = 1 + 2 + \dots + n$ then [EAMCET 2007]

1) $S_n = T_n^3$ 2) $S_n = T_n^2$ 3) $S_n = T_n^5$ 4) $S_n = T_n^4$

Ans: 2

Sol. $S_n = \sum n^3 = \frac{n^2(n+1)^2}{4} = (\sum n)^2 = T_n^2$

4. For all integers $n \geq 1$, which of the following is divisible by 9 [EAMCET 2006]

1) $8^n + 1$ 2) $4^n - 3n - 1$ 3) $3^{2n} + 3n + 1$ 4) $10^n + 1$

Ans: 2

Sol. by verification $n = 2$

5. $\{n(n+1)(2n+1) : n \in \mathbb{Z}\} \subset$ [EAMCET 2005]

1) $\{6k : k \in \mathbb{Z}\}$ 2) $\{12k : k \in \mathbb{Z}\}$ 3) $\{18k : k \in \mathbb{Z}\}$ 4) $\{24k : k \in \mathbb{Z}\}$

Ans: 1

Sol. $n(n+1)(2n+1)$

$= 6 \cdot \frac{n(n+1)(2n+1)}{6} = 6k, k \in \mathbb{Z} \left[\because \sum n^2 \text{ is an integer} \right]$

6. $\sum_{k=1}^5 \frac{1^3 + 2^3 + \dots + k^3}{1 + 3 + 5 + \dots + (2k-1)}$ [EAMCET 2004]

1) 22.5 2) 24.5 3) 28.5 4) 32.5

Ans: 1

Sol. $\sum_{k=1}^5 \frac{k^2(k+1)^2}{4k^2}$

$$= \frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{4} = 22.5$$

7. If $t_1 = \frac{1}{4}(n+2)(n+3)$ for $n = 1, 2, \dots$ then $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \dots + \frac{1}{t_{2003}} =$ **[EAMCET 2003]**

- 1) $\frac{4006}{3006}$ 2) $\frac{4003}{3007}$ 3) $\frac{4006}{3008}$ 4) $\frac{4006}{3009}$

Ans: 4

Sol. $\frac{1}{t_n} = \frac{4}{(n+2)(n+3)}$
 $= 4 \left[\frac{1}{n+2} - \frac{1}{n+3} \right]$
 $\therefore \frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n}$
 $4 \left[\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{n+2} - \frac{1}{n+3} \right]$
 $= \frac{4n}{3(n+3)} = \frac{4(2003)}{3(2006)} = \frac{4006}{3009}$

8. In the sequence $\{1\}, \{2, 3\}, \{4, 5, 6\}, \{7, 8, 9, 10\}, \dots$ of sets the sum of elements in the 50th set is **[EAMCET 2002]**

- 1) 62525 2) 65225 3) 56255 4) 55625

Ans: 1

Sol. The sum of elements in the nth set is

$$S_n = \frac{n(n^2+1)}{2}$$

$$\therefore S_{50} = \frac{50(50^2+1)}{2} = 62525$$

9. If $a_k = \frac{1}{k(k+1)}$, for $k = 1, 2, 3, \dots, n$ then $\left(\sum_{k=1}^n a_k \right)^2 =$ **[EAMCET 2000]**

- 1) $\frac{n}{n+1}$ 2) $\frac{n^2}{(n+1)^2}$ 3) $\frac{n^4}{(n+1)^4}$ 4) $\frac{n^6}{(n+1)^6}$

Ans: 1

Sol. Given $a_k = \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$
 $= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \left[\frac{1}{2} + \frac{1}{3} + \dots \right]$
 $\frac{1}{k+1} = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$

