

LIMITS

PREVIOUS EAMCET BITS

1. $\lim_{x \rightarrow \infty} \left(\frac{x+5}{x+2} \right)^{x+3} =$ [EAMCET 2009]

- 1) e 2) e^2 3) e^3 4) e^5

Ans: 3

Sol. $e^{\lim_{x \rightarrow \infty} (x+3) \left[\frac{x+5}{x+2} - 1 \right]} = e^{\lim_{x \rightarrow \infty} \frac{3x+9}{x+2}} = e^3 \left[\because \lim_{x \rightarrow \infty} [f(x)]^{g(x)} = e^{\lim_{x \rightarrow \infty} g(x)[f(x)-1]} \right]$

2. $\lim_{x \rightarrow 0} \frac{(1-e^x) \sin x}{x^2 + x^3}$ [EAMCET 2008]

- 1) -1 2) 0 3) 1 4) 2

Ans: 1

Sol. $\lim_{x \rightarrow 0} \frac{(1-e^x) \sin x}{x^2 + x^3} = \lim_{x \rightarrow 0} \frac{1-e^x}{x+x^2} \times \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{1-e^x}{x+x^2} = \lim_{x \rightarrow 0} \frac{-e^x}{1+2x} = -1$

3. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = [x-3] + |x-4|$ for $x \in \mathbb{R}$ then $\lim_{x \rightarrow 3^-} f(x) =$ [EAMCET 2008]

- 1) -2 2) -4 3) -6 4) -8

Ans: 3

Sol. $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} -\{[x-3] + |x-4|\} = -1+1 = 0$

4. If $f(2) = 4$ and $f'(2) = 1$ then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2} =$ [EAMCET 2008]

- 1) -2 2) 1 3) 2 4) 3

Ans: 3

Sol. $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2} = \lim_{x \rightarrow 2} \frac{f(2) - 2f'(2)}{1} = \frac{4-2}{1} = 2$

5. $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{2(x - \sin x)} =$ [EAMCET 2007]

- 1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{3}{2}$

Ans: 2

Sol. $\lim_{x \rightarrow 0} \frac{e^{\sin x} (e^{x-\sin x} - 1)}{2(x - \sin x)} = \frac{1}{2}$

6. If $f(x) = \begin{cases} \frac{\sin(1+[x])}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer not exceeding x , then

$\lim_{x \rightarrow 0^-} f(x) =$ [EAMCET 2007]

- 1) -1 2) 0 3) 1 4) 2

Ans: 2

Sol. $\lim_{x \rightarrow 0^-} f(x)$
 $= \lim_{x \rightarrow 0^-} \frac{\sin(1 + [x])}{[x]} \times \frac{1 + [x]}{1 + [x]}$
 $= 1 \times \frac{0}{-1} = 0$

7. If $0 < p < q$, then $\lim_{n \rightarrow \infty} (q^n + p^n)^{1/n} =$ **[EAMCET 2006]**

- 1) e 2) p 3) q 4) 0

Ans: 3

Sol. $0 < p < q, 0 < \frac{p}{q} < 1$

$$\lim_{n \rightarrow \infty} (q^n + p^n)^{1/n} = \lim_{n \rightarrow \infty} q \left(1 + \frac{p}{q}\right)^{1/n} = q \times 1 = q$$

8. $\lim_{n \rightarrow \infty} [\sqrt{x^2 + 2x - 1} - x] =$ **[EAMCET 2006]**

- 1) ∞ 2) $1/2$ 3) 4 4) 1

Ans: 4

Sol. $\lim_{x \rightarrow \infty} \frac{2x - 1}{\sqrt{x^2 + 2x - 1} + x}$

$$\lim_{x \rightarrow \infty} \frac{\left(2 - \frac{1}{x}\right)}{\sqrt{1 + \frac{2}{x} - \frac{1}{x^2}} + 1} = \frac{2}{2} = 1$$

9. If $\lim_{x \rightarrow 0} \left(\frac{\cos 4x + a \cos 2x + b}{x^4}\right)$ is finite, then the values of a, b, are respectively **[EAMCET 2006]**

- 1) 5, -4 2) -5, -4 3) -4, 3 4) 4, 5

Ans: 3

Sol. $\lim_{x \rightarrow 0} \left(\frac{\cos 4x + a \cos 2x + b}{x^4}\right) = \lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ exists

$$f(0) = 0$$

$$1 + a + b = 0 \Rightarrow a + b = -1$$

$$\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 0} \frac{-4 \sin 4x - 2a \sin 2x}{4x^3}$$

$$f''(0) = 0$$

$$-16 - 4a = 0 \Rightarrow a = -4$$

$$\Rightarrow b = 4 - 1 = 3 \quad -4, 3$$

10. If $I_1 = \lim_{x \rightarrow 2^+} (x + [x]), I_2 = \lim_{x \rightarrow 2^-} (2x + [x])$ and $I_3 = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\left(x - \frac{\pi}{2}\right)}$, then **[EAMCET 2006]**

- 1) $I_1 < I_2 < I_3$ 2) $I_2 < I_3 < I_1$ 3) $I_3 < I_2 < I_1$ 4) $I_1 < I_3 < I_2$

Ans: 3

Sol. $l_1 = \lim_{x \rightarrow 2^+} x + [x] = 4$
 $l_2 = \lim_{x \rightarrow 2^-} 2x - [x]$
 $= \lim_{h \rightarrow 0} \{2(2-h) - [2-h]\}$
 $= 4 - 2 = 2$

$$l_3 = (-1) \lim_{\left(\frac{\pi}{2}-x\right)} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)} = -1$$

$$l_3 < l_2 < l_1$$

11. $\lim_{x \rightarrow 0} x^2 \sin \frac{\pi}{x} =$ [EAMCET 2005]

- 1) 1 2) 0 3) does not exist 4) ∞

Ans: 2

Sol. $\lim_{x \rightarrow 0} x^2 \sin \left(\frac{\pi}{x}\right) = \lim_{x \rightarrow 0} x^2 \cdot \left(\lim_{x \rightarrow 0} \sin \frac{\pi}{x}\right) = 0 \times (\text{finite value between } -1 \text{ to } 1) = 0$

12. $\lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n (k^2 x) =$ [EAMCET 2004]

- 1) x 2) $\frac{x}{2}$ 3) $\frac{x}{3}$ 4) $\frac{x}{4}$

Ans: 3

Sol. $\lim_{n \rightarrow \infty} \frac{1}{n^3} \left[\frac{n(n+1)(2n+1)}{6} \right] = \lim_{n \rightarrow \infty} \frac{2n^3 + \dots}{6n^3} = \frac{x}{6} \times \frac{2}{6} = \frac{x}{3}$

13. $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{3 \sin x - \sqrt{3} \cos x}{6x - \pi} \right) =$ [EAMCET 2003]

- 1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $-\frac{1}{\sqrt{3}}$ 4) $\frac{-1}{\sqrt{3}}$

Ans: 2

Sol. $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{3 \sin x - \sqrt{3} \cos x}{6x - \pi} \right)$
 $\lim_{x \rightarrow \frac{\pi}{6}} \frac{3 \cos x + \sqrt{3} \sin x}{6} = \frac{1}{\sqrt{3}}$

14. If $a > 0 \lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$, then $a = \dots\dots\dots$ [EAMCET 2003]

- 1) 0 2) 1 3) e 4) 2e

Ans: 2

Sol. $\lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$

Apply L-hospital rule

$$\Rightarrow \lim_{x \rightarrow a} \frac{a^x \log a - ax^{a-1}}{x^x(1 + \log x)} = -1$$

$$\Rightarrow \frac{\log a - 1}{1 + \log a} = -1 \Rightarrow \log a = 0 \Rightarrow a = 1$$

15. $\lim_{x \rightarrow 0} \frac{4^x - 9^x}{x(4^x + 9^x)} =$ **[EAMCET 2002]**

1) $\log \frac{2}{3}$ 2) $\log \frac{3}{2}$ 3) $\frac{1}{2} \log \frac{2}{3}$ 4) $\frac{1}{2} \log \frac{3}{2}$

Ans: 1

Sol. $\lim_{x \rightarrow 0} \frac{4^x - 9^x}{x(4^x + 9^x)}$

$$\frac{(4^x - 1) - (9^x - 1)}{x(4^x + 9^x)}$$

$$\lim_{x \rightarrow 0} \frac{x - x}{4^x + 9^x}$$

$$\frac{\log 4 - \log 9}{2} = \frac{1}{2} \log \left(\frac{2}{3} \right)^2 = \log \frac{2}{3}$$

16. The quadratic equation whose roots are l and m where

$$l = \lim_{\theta \rightarrow 0} \left(\frac{3 \sin \theta - 4 \sin^2 \theta}{\theta} \right) \text{ and } m = \lim_{\theta \rightarrow 0} \left(\frac{2 \tan \theta}{\theta(1 - \tan^2 \theta)} \right) \text{ is}$$
 [EAMCET 2002]

1) $x^2 + 5x + 6 = 0$ 2) $x^2 - 5x + 6 = 0$ 3) $x^2 - 5x - 6 = 0$ 4) $x^2 + 5x - 6 = 0$

Ans: 2

Sol. $l = \lim_{\theta \rightarrow 0} \left(\frac{3 \sin \theta - 4 \sin^2 \theta}{\theta} \right)$

$$= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{3\theta} (3 - 4 \sin \theta) = 3$$

$$m = \lim_{\theta \rightarrow 0} \frac{2 \tan \theta}{\theta(1 - \tan^2 \theta)} = 2$$

\therefore The quadratic equation required is $x^2 - (l + m)x + lm = 0$

$$x^2 - (3 + 2)x + 6 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

17. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x - [x]$, where $[x]$ is the greatest integer not exceeding x , then the set of discontinuities of f is **[EAMCET 2002]**

1) the empty set 2) \mathbb{R} 3) \mathbb{Z} 4) \mathbb{N}

Ans: 3

Sol. $[x]$ is discontinuous at inter values of x hence $f(x) = x - [x]$ is discontinuous on \mathbb{Z}

18. $\lim_{x \rightarrow \infty} \left(\frac{x+a}{x+b} \right)^{x+b} =$ **[EAMCET 2001]**

1) 1 2) e^{b-a} 3) e^{a-b} 4) e^b

Ans: 3

$$\text{Sol. } e^{\text{Lt}_{x \rightarrow \infty} (x+b) \left[\frac{x-a}{x+b} - 1 \right]} = e^{a-b} \left[\because \text{Lt}_{x \rightarrow \infty} f[x]^{g(x)} = e^{\text{Lt}_{x \rightarrow \infty} g(x)[f(x)-1]} \right]$$

$$19. \lim_{x \rightarrow \alpha} \frac{x \cdot 10^x - x}{1 - \cos x} =$$

[EAMCET 2001]

1) $\log 10$

2) $2 \log 10$

3) $3 \log 10$

4) $4 \log 10$

Ans: 2

$$\text{Sol. } \lim_{x \rightarrow 0} \frac{x(10^x - 1)}{2 \sin^2 \frac{x}{2}}$$

$$\lim_{x \rightarrow 0} \left(\frac{10^x - 1}{x} \right) \cdot \lim_{x \rightarrow 0} \frac{x^2}{2 \sin^2 \frac{x}{2}} = 2 \log 10$$

$$20. \text{ If } f(x) = \frac{x^2 - 10x + 25}{x^2 - 7x + 10} \text{ for } x \neq 5 \text{ and } f \text{ is continuous at } x = 5 \text{ then } f(5) =$$

[EAMCET 2001]

1) 0

2) 5

3) 10

4) 25

Ans: 1

$$\text{Sol. } \lim_{x \rightarrow 5} \frac{x^2 - 10x + 25}{x^2 - 7x + 10} = f(5)$$

$$\Rightarrow f(5) = 0$$

$$21. \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\cos \theta \left(\frac{\pi}{2} - \theta \right)} =$$

[EAMCET 2000]

1) 1

2) -1

3) $-\frac{1}{2}$

4) $\frac{1}{2}$

Ans: 4

$$\text{Sol. } \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\cos \theta \left(\frac{\pi}{2} - \theta \right)}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{2 \sin^2 \left(\frac{\pi - \theta}{2} \right)}{2 \sin \left(\frac{\pi - \theta}{2} \right) \cos \left(\frac{\pi - \theta}{2} \right) \cdot 2 \left(\frac{\pi - \theta}{2} \right)} = \frac{1}{2}$$

$$22. \lim_{x \rightarrow 0} \frac{\log_e^{(x+1)}}{3^x - 1} =$$

[EAMCET 2000]

1) \log_e^3

2) 0

3) 1

4) \log_3^e

Ans: 4

Sol. $\lim_{x \rightarrow 0} \frac{\log_e^{(1+x)}}{3^x - 1}$

$$\lim_{x \rightarrow 0} \frac{\log_e^{(1+x)}}{\left(\frac{3^x - 1}{x}\right)} = \frac{\log_e^e}{\log_e^3} = \log_3^e$$



नाम