

HYPERBOLIC FUNCTIONS

PREVIOUS EAMCET BITS

1. $\sinh^{-1} 2 + \sinh^{-1} 3 = x \Rightarrow \cosh x =$ [EAMCET 2009]

$$1) \frac{1}{2}(3\sqrt{5} + 2\sqrt{10}) \quad 2) \frac{1}{2}(3\sqrt{5} - 2\sqrt{10}) \quad 3) \frac{1}{2}(12 + 2\sqrt{50}) \quad 4) \frac{1}{2}(12 - 2\sqrt{50})$$

Ans: 3

Sol. $\cosh x = \cosh(\sinh^{-1} 2 + \sinh^{-1} 3)$

$$= \frac{1}{2}(12 + 2\sqrt{50})$$

2. $\frac{1 + \tanh(x/2)}{1 - \tanh(x/2)}$ [EAMCET 2008]

$$1) e^{-x} \quad 2) e^x \quad 3) 2e^{x/2} \quad 4) 2e^{-x/2}$$

Ans: 2

Sol.
$$\begin{aligned} \frac{1 + \tanh\left(\frac{x}{2}\right)}{1 - \tanh\left(\frac{x}{2}\right)} &= \frac{\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - \sinh\left(\frac{x}{2}\right)} = \frac{\left[\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)\right]^2}{\cosh^2\left(\frac{x}{2}\right) - \sinh^2\left(\frac{x}{2}\right)} \\ &= \cosh^2\left(\frac{x}{2}\right) + \sinh^2\left(\frac{x}{2}\right) + 2\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right) = \cosh x + \sinh x = e^x \end{aligned}$$

3. $\operatorname{sech}^{-1}(\sin \theta) =$ [EAMCET 2007]

$$1) \log \tan \frac{\theta}{2} \quad 2) \log \sin \frac{\theta}{2} \quad 3) \log \cos \frac{\theta}{2} \quad 4) \log \cot \frac{\theta}{2}$$

Ans: 4

Sol. $\operatorname{sech}^{-1}(\sin \theta) = \log\left(\frac{1 + \sqrt{1 - \sin^2 \theta}}{\sin \theta}\right)$

$$= \log\left(\frac{1 + \cos \theta}{\sin \theta}\right) = \log(\cot \theta / 2)$$

4. $e^{\log(\cosh^{-1} 2)} =$ [EAMCET 2006]

$$1) \log(2 - \sqrt{3}) \quad 2) \log(\sqrt{3} - 2) \quad 3) \log(2 + \sqrt{3}) \quad 4) \log(2 + 5)$$

Ans: 3

Sol. $e^{\log_e f(x)} = f(x)$

$$e^{\log_e \cosh^{-1}(2)} = \cosh^{-1}(2)$$

$$= \log\left\{2 + \sqrt{2^2 - 1}\right\}$$

$$= \log(2 + \sqrt{3})$$

5. $2 \tanh^{-1} \frac{1}{2} =$ [EAMCET 2005]

- 1) 0 2) $\log 2$ 3) $\log 3$

Ans: 3

Sol. $\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{1-x}$

$$\therefore 2 \tanh^{-1} x = \log \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \log 3$$

6. $x = \log \left[\cot \left(\frac{\pi}{4} + \theta \right) \right] \Rightarrow \sinh x =$ [EAMCET 2004]

- 1) $\tan 2\theta$ 2) $-\tan 2\theta$ 3) $\cot 2\theta$ 4) $-\cot 2\theta$

Ans: 2

Sol. $\sinh x = \frac{\cot \left(\frac{\pi}{4} + \theta \right) - \tan \left(\frac{\pi}{4} + \theta \right)}{2} = -\tan 2\theta$

7. $\sinh^{-1} (2^{3/2}) = \dots$ [EAMCET 2003]

- 1) $\log(2 + \sqrt{18})$ 2) $\log(3 + \sqrt{8})$ 3) $\log(3 - \sqrt{8})$

Ans: 2

Sol. $\sinh^{-1} (2^{3/2}) = \log(\sqrt{8} + \sqrt{1+8}) = \log(3 + \sqrt{8})$

8. $\sin h ix =$ [EAMCET 2002]

- 1) $i \sin x$ 2) $\sin(ix)$ 3) $-\sin x$

Ans: 1

Sol. $\sinh(ix) = \frac{e^{ix} - e^{-ix}}{2} = \frac{(\cos x + i \sin x) - (\cos x - i \sin x)}{2} = i \sin x$

9. $\sec^2(\tan^{-1} 2) + \csc^2(\cot^{-1} 3) =$ [EAMCET 2001]

- 1) 5 2) 10 3) 15 4) 20

Ans: 3

Sol. Let $\tan^{-1}(2) = \alpha$ and $\cot^{-1}(3) = \beta$

$\tan \alpha = 2$; $\cot \beta = 3$

$\Rightarrow \sec \alpha = \sqrt{5}$; $\csc \beta = \sqrt{10}$

$\therefore \sec^2 \alpha + \csc^2 \beta = 5 + 10 = 15$

10. $\cosh 2 + \sinh 2 =$ [EAMCET 2000]

- 1) $1/e$ 2) e 3) $1/e^2$ 4) e^2

Ans: 4

Sol. $\cosh 2 + \sinh 2 = \frac{e^2 + e^{-2}}{2} + \frac{e^2 - e^{-2}}{2} = e^2$

11. If $\cosh^{-1} x = \log_e(2 + \sqrt{3})$, then $x =$ [EAMCET 2000]

- 1) 2 2) 1 3) 3 4) 5

Ans: 1

Sol. $\log\left[x + \sqrt{x^2 - 1}\right] = \log(2 + \sqrt{3})$

$$\log\left[x + \sqrt{x^2 - 1}\right] = \log\left[2 + \sqrt{(2)^2 - 1}\right]$$

$$\therefore x = 2$$

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